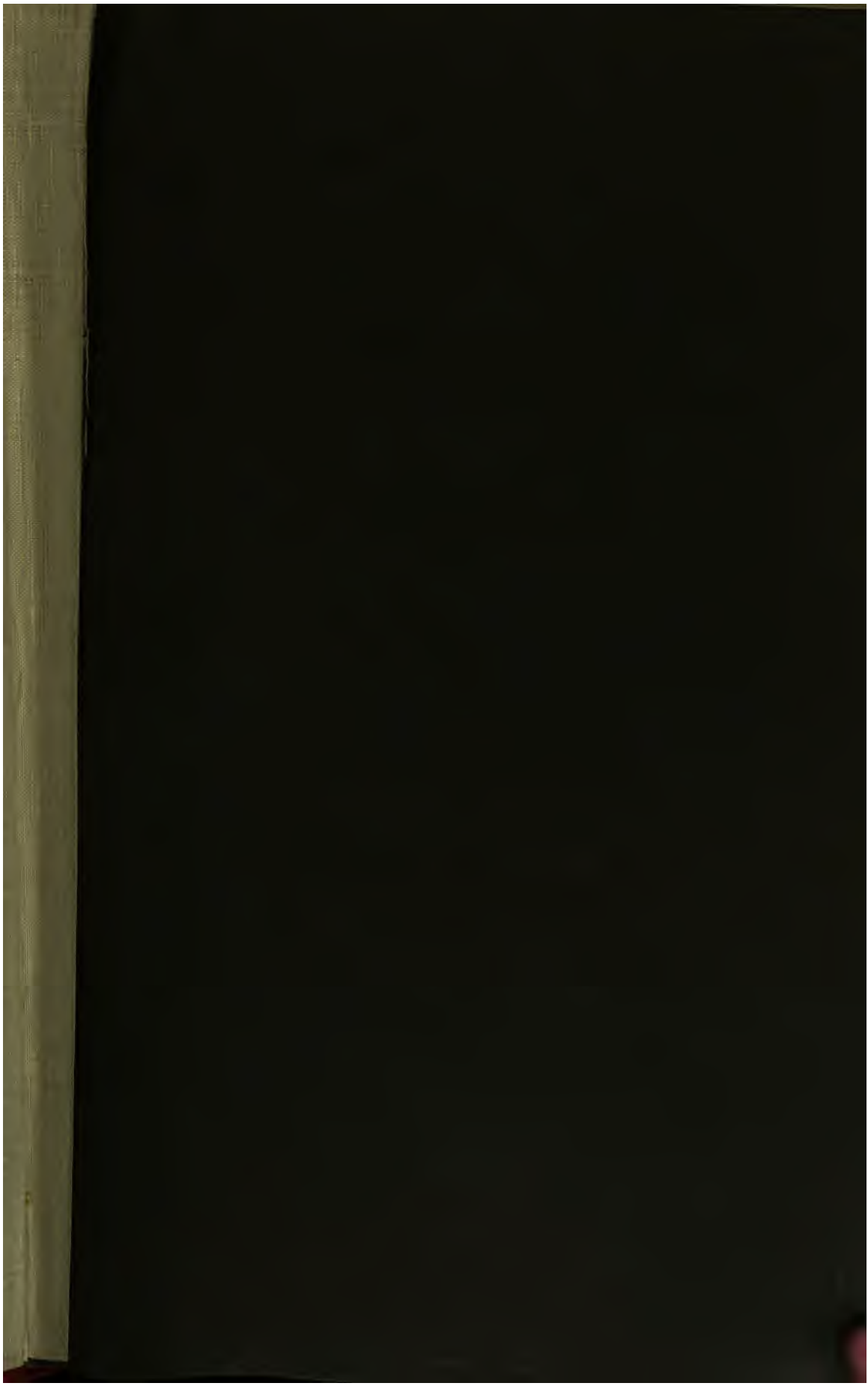


A. K. Mowbray

July 28th 1907.

BRITISH OFFICES LIFE TABLES, 1893.

AN

ACCOUNT

OF THE

PRINCIPLES AND METHODS

ADOPTED IN THE

COMPILATION OF THE DATA,

THE

GRADUATION OF THE EXPERIENCE

AND THE

CONSTRUCTION OF DEDUCED TABLES.

ASSURED LIVES

AND

LIFE ANNUITANTS.

PREPARED AND PUBLISHED ON THE AUTHORITY AND UNDER THE SUPERINTENDENCE OF

THE INSTITUTE OF ACTUARIES

AND

THE FACULTY OF ACTUARIES IN SCOTLAND.

LONDON:
CHARLES AND EDWIN LAYTON,
56, FARRINGDON STREET, E.C.

1903.

INTRODUCTION.

(1). The present volume completes the series proposed to be published by the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in Scotland on Mortality Investigation. It contains a series of detailed accounts of the technical processes and methods adopted by the Committee, and in this Introduction the opportunity has been taken of making a statement as to the general conduct of the investigation.

(2). The investigation was commenced in the year 1893, and has since 1897, been conducted by Committees of the Institute and Faculty, respectively designated the "London Section" and the "Scottish Section." Meetings of these Sections were held separately in London and Edinburgh for preliminary discussion; and combined meetings of the two Sections were held in London to discuss and decide the matters of principle which arose from time to time in the progress of the work.

(3). The Joint Committee was constituted as follows:—

CHAIRMAN OF COMMITTEE.

RALPH PRICE HARDY.

LONDON SECTION.

*THOMAS GANS ACKLAND.	GEORGE KING.
ARTHUR FRANCIS BURRIDGE.	*GEORGE JAMES LIDSTONE.
HENRY COCKBURN.	HENRY WILLIAM MANLY.
†ALEXANDER JOHN FINLAISON, C.B.	GERALD HEMMINGTON RYAN.
GEORGE FRANCIS HARDY.	FREDERICK SCHOOLING.
RALPH PRICE HARDY.	WILLIAM JOSEPH H. WHITTALL.
CHARLES DANIEL HIGHAM.	FRANK BERTRAND WYATT.
*WILLIAM HUGHES.	THOMAS EMLEY YOUNG, B.A.

SCOTTISH SECTION.

DAVID DEUCHAR.	JAMES MEIKLE.
*GORDON DOUGLAS.	THOMAS BOND SPRAGUE, M.A., LL.D.
*NIEL BALLINGAL GUNN.	SPENCER CAMPBELL THOMSON, B.A.
GEORGE MACRITCHIE LOW.	ANDREW HUGH TURNBULL.

GORDON DOUGLAS, *Hon. Secretary of Scottish Section.*

A. F. BURRIDGE, THOMAS G. ACKLAND, *Hon. Secretaries of London Section, and of Joint Committee.*

* Elected in the course of the investigation.
† Died " " "

(4). The following Joint Letter was sent out in December, 1893, to all those Companies whose experience, in respect of Assured Lives or Annuitants, it was thought desirable to include:—

INSTITUTE OF ACTUARIES,
STAPLE INN HALL, HOLBORN,
LONDON, W.C.

FACULTY OF ACTUARIES IN SCOTLAND,
24, YORK PLACE, EDINBURGH.

DEAR SIR,

NEW MORTALITY EXPERIENCE.

It has been resolved to attempt the compilation of a new collective Mortality Experience of Assured Lives, and separately of Annuity Nominees, in the United Kingdom. The most recent Tables of the kind as regards Assured lives were formed upon observations which terminated thirty years ago, while the Annuity tables comprise only the Government Experience. Changes have taken place in the condition of the population, which render it by no means improbable that the rate of Mortality is not now the same as formerly. The Institute of Actuaries' Tables, valuable as they have proved, were based on an experience which, compared with the amount of material now accumulated in the records of the Offices, must be considered very limited. Moreover, the Institute of Actuaries' Tables included the experience of Companies over a long period of time, dating from the early years of the present century, and it is therefore thought to be of great importance that new tables should be compiled relating to the experience of the present generation. It is also generally felt that a table might now be constructed which would more satisfactorily exhibit the characteristics of mortality among assured lives, as influenced by initial selection and the subsequent duration of the policies.

The Council of the Institute of Actuaries and the Council of the Faculty of Actuaries will co-operate in this work.

It is earnestly desired that as many Companies as possible shall contribute their experience. At a later stage we shall have to communicate with you as to the means of defraying the expense of an undertaking so important to all Assurance Companies; but, independently of that question, our object at present is to secure material for the investigation. We shall esteem it a favour, therefore, if you will kindly submit the question to your Directors, and ascertain whether your Company will join in the investigation. It is proposed to obtain the experience for the period from 1863 to 1893 (including, of course, lives on the books in 1863, and new entrants since that date), and to confine it to lives accepted at the ordinary rate of premium. No doubt, when the materials have been received from the Companies, it will be found possible to carry out various subsidiary investigations.

The particulars which it will be very desirable to obtain regarding each Assured Life—and each Annuitant as far as applicable—are as follows:—

1. No. of Policy.
2. Sum Assured.
3. Description of Policy.
4. Profit or Non-profit.
5. Name of Life.
6. Occupation.
7. Date of Birth.
8. Date of Entry.
9. Date of Exit.
10. Mode of Exit.

Be good enough to say whether, in the event of your Company contributing its experience, these particulars can be supplied, or, if not, what is the nearest approach that can be given to them.

When it has been ascertained which Companies will contribute, a letter of detailed explanations will be issued, together with the forms of card which will be used.

We remain,

Yours faithfully,

AUGUSTUS HENDRIKS,
President of the Institute of Actuaries.

JAMES MEIKLE,
President of the Faculty of Actuaries.

(5.) In reply to this letter, 66 Companies expressed their willingness to contribute to the experience. In the list on pages vi and vii are given the names and dates of establishment of each contributing Company, the nature of the data furnished being indicated by an asterisk in the appropriate column. It will be seen that 60 Offices contributed to the experience of assured lives, and 43 to the experience of annuitants.

(6). Instructions were issued to the Offices, in May and August 1894, as to the plan upon which it was desired that the data as to Assured Lives and Annuitants should be taken out and supplied. Copies of these instructions are given for Annuitants on pages 22 to 24, and for Assured Lives on pages 81 to 85 of the present volume. Some questions having subsequently arisen as to the interpretation of these instructions, a supplementary statement was issued to the Offices in April, 1895, dealing with these points of enquiry. A copy of this supplementary statement is given on page 86 of the present volume.

Companies contributing through the Institute of Actuaries.

Name of Company.	Estd.	Assurances.	Annuities.
Alliance	1824	*	—
Atlas	1808	*	—
British Empire Mutual	1847	*	*
Clergy Mutual	1829	*	—
Clerical, Medical and General	1824	*	—
Commercial Union...	1861	*	*
Eagle	1807	*	—
Economic	1823	*	—
English and Scottish Law †	1839	*	*
Equitable	1762	*	—
Equity and Law	1844	*	*
Friends' Provident	1832	*	*
General	1837	*	*
Gresham	1848	*	*
Guardian	1821	*	*
Hand-in-Hand	1696	*	*
Imperial and England	1820	*	*
	1840	*	*
Lancashire	1852	*	—
Law Life	1823	*	—
Law Union and Crown	1825	*	*
Legal and General	1836	*	*
Liverpool and London and Globe ...	1836	*	*
London and Lancashire	1862	*	—
London Assurance	1720	*	*
London Life Association	1806	*	—
Metropolitan	1835	*	—
Mutual	1834	*	—
National	1830	*	*
National of Ireland	1822	—	*
National Provident...	1835	*	*
North British and Mercantile †	1823	*	*
Norwich Union	1808	*	*
Patriotic	1824	*	—
Pelican	1797	*	—
Provident	1806	*	—
Provident Clerks'	1840	*	*
Prudential	1848	—	*
Rock	1806	*	—
Royal	1845	*	*
Royal Exchange	1720	*	*
Sun	1810	*	—
Union	1714	*	—
United Kent	1824	*	*
United Kingdom Temperance and General	1840	—	*
Universal	1834	*	—
University	1825	*	—
Westminster and General	1836	*	*
Yorkshire	1823	*	*

† These Companies contributed their English and Scottish business respectively through the Institute and the Faculty.

The following American Offices contributed their British Annuity Experience only:—

Name of Company.	Estd.	Assurances.	Annuities.
Equitable of the United States	1859	—	*
New York Life	1845	—	*
Mutual of New York	1843	—	*

Companies Contributing through the Faculty of Actuaries.

Name of Company.	Estd.	Assurances.	Annuities.
Caledonian	1805	*	*
City of Glasgow	1838	*	*
Edinburgh	1823	*	*
English and Scottish Law †	1839	*	*
Life Association of Scotland	1838	*	*
North British and Mercantile †	1823	*	*
Northern	1836	*	*
Scottish Amicable	1826	*	*
Scottish Equitable	1831	*	—
Scottish Imperial	1865	*	—
Scottish Life	1881	*	*
Scottish Metropolitan	1876	*	*
Scottish Provident	1837	*	*
Scottish Union and National	1824	*	*
Scottish Widows' Fund	1815	*	*
Standard	1825	*	*

† These Companies contributed their English and Scottish business respectively through the Institute and the Faculty.

(7). The cards required for the record of the data in respect of individual policies were furnished to the contributing Offices in December, 1894, and January, 1895. In the case of each English Office, a special number, and in the case of each Scottish Office, a distinctive letter, was printed at the foot of the cards, so that each Company was represented, so far as the general body of workers was concerned, by a symbol only. Specimens of the cards employed for the Annuitants are given in the present volume on page 34, and for Assured Lives on pages 56, 57.

(8). The completed cards relating to the Annuitant Experience were received from the contributing English and Scottish Offices towards the end of the year 1896, and those relating to Assured Lives were all sent in by March, 1897.

(9). The task of compilation and arrangement of the data was one of great magnitude and delicacy. By the courtesy of the

Institute of Actuaries, the Hall and Class Rooms of Staple Inn were placed at the disposal of the Committee, and a staff of clerks, varying in number from six to thirty-five, was employed daily, from July, 1896, to August, 1900, in arranging and tabulating the cards on which the data were supplied. This staff was under the honorary supervision of Mr. THOMAS G. ACKLAND, F.I.A., and it is due to his unwearied attention to details, and his unfailing skill in statistical processes, that the work has been brought to a successful issue.

(10). The elimination of duplicates, and the settlement of the numerous inevitable queries, being disposed of, the work of tabulation was finally completed in August, 1900, and the volumes showing the unadjusted data were published in the following order:—

(1). Life Annuitants—Male and Female. (January, 1899).

(2). Endowment Assurances, and Minor Classes of Assurance—Male and Female. The Minor Classes comprise Whole Life Assurances with limited payments, and with ascending premiums; Joint Life Assurances; Contingent Survivorship Assurances; and Temporary Assurances. (January, 1900).

(3). Whole Life Assurances—Male Lives—Participating and Non-Participating. (July, 1900).

(4). Whole Life Assurances—Female Lives—Participating and Non-Participating. (November, 1900).

A general summary of the data included in the whole experience comprised in these four volumes is given on page xii.

(11). The principles and methods followed in the compilation of the data in the different sections of the experience are set out in detail on pages 1 to 120 of the present volume, by Mr. T. G. ACKLAND, the Hon. Supervisor of the work.

(12). The Committee consider themselves singularly fortunate in having been able to place the work of graduation in the hands of Mr. G. F. HARDY, F.I.A. Readers of this volume, and those who make use of the Tables, will need no reminder of the pre-eminent position which Mr. HARDY occupies in connection with the subject of graduation, but the Committee desire here to place on record their high appreciation of the manner in which he has placed his unrivalled skill and knowledge at their disposal. An account of the principles and methods adopted is given by him on pages 121 to 166 of the present volume.

(13). The computation of the elementary mortality values for Life Annuitants, and the mortality and monetary tables for Assured Lives, was undertaken by Mr. H. J. BAKER, F.I.A., with the assistance of a competent London Staff. The monetary tables in respect of Life Annuitants for single and joint lives were computed in Edinburgh under the honorary supervision of Mr. JAMES CHATHAM, F.I.A., F.F.A. Accounts of the methods followed in the calculations (so far as these are not upon lines generally followed) have been prepared by Mr. BAKER and Mr. CHATHAM, and will be found on pages 167 to 176, and pages 177 to 185 of the present volume respectively.

(14.) The three volumes setting forth the graduated tables and the monetary values were published at the dates specified :—

BRITISH OFFICES LIFE ANNUITY TABLES :—

(1) Select Tables, Male Lives—$O^{[am]}$—, Female Lives—$O^{[af]}$—. (December, 1902).

BRITISH OFFICES LIFE TABLES :—

Whole Life Participating Assurances—Males.

(2) Aggregate Tables—O^{M}—, and Aggregate Tables excluding the first 5 years—$O^{M(5)}$—.(May, 1902).

(3) Select Tables—$O^{[M]}$—.(June, 1903).

(15.) The Committee recognise that the tables included in these published volumes represent a portion only of those which might usefully be computed to meet fully the practical needs of the profession. It is hoped, however, that the Tables published under their authority will be found to furnish an important contribution towards those needs; and that they may, from time to time, be supplemented by the skilled labours of individual members (and especially of the younger members) of the profession.

(16.) It was felt that, for the convenience of the profession, and generally of those who employ, or have occasion to cite the published Tables, it was desirable to adopt an authoritative set of official designations and symbols applicable to the new Experience, which is printed on page xi.

(17.) It is hardly necessary to say that a work of this magnitude, requiring, for a great part of the time, the employment of a numerous staff, involved a very considerable outlay. The Committee thankfully recognise the liberal response which has been made by the Life Assurance Companies to their appeal for the necessary funds.

(18.) Upon the constitution of the Joint Committee in 1897, Mr. F. B. WYATT, and, on his resignation in July, 1898, Mr. THOMAS G. ACKLAND, were associated with Mr. A. F. BURRIDGE as Joint Honorary Secretaries of the Committee, and of the London Section; whilst Mr. GORDON DOUGLAS has rendered valuable service as Honorary Secretary of the Scottish Section. The Joint Committee most gratefully acknowledge the zealous co-operation of these gentlemen; and also desire to acknowledge the assistance they have received, not only from those whose services have already been specifically recognised, but from every member of the staff, and from many others, not formally engaged on the investigation, who have from time to time contributed useful information and suggestions.

(19.) And they finally close this undertaking with the expression of the hope that, though the primary intention of the investigation was for the benefit of Life Assurance Companies and kindred institutions, it may also be found useful in assisting the researches of students of Economics in the many problems where vital statistics are concerned, and may thus contribute towards the general benefit of mankind.

WM. HUGHES,
President of the Institute of Actuaries.

NIEL B. GUNN.
President of the Faculty of Actuaries in Scotland.

October, 1903.

JOINT MORTALITY INVESTIGATION.

OFFICIAL DESIGNATIONS AND SYMBOLS.

(Adopted by Joint Committee, June, 1903.)

THE BRITISH OFFICES LIFE TABLES, 1893.

FULL AGGREGATE TABLE—MALES:—

Whole-Life Participating Assurances... ...	O^{M}
Whole-Life Non-Participating Assurances ...	O^{NM}
Endowment Assurances	O^{EM}
Assurances with Limited Payments	O^{LM}
Assurances with Increasing Scale of Premiums	O^{IM}
Temporary Assurances...	O^{TM}
Contingent Assurances	O^{CM}
Joint Assurances	O^{JM}

"TRUNCATED" AGGREGATE TABLES—MALES:—

Whole-Life Participating Assurances (excluding the first five years of Assurance)	$O^{M(5)}$
&c., &c., &c., &c.	&c.

SELECT TABLES—MALES:—

Whole-Life Participating Assurances... ...	$O^{[M]}$
&c., &c., &c.	&c.

THE BRITISH OFFICES LIFE ANNUITY TABLES, 1893.

Male Annuitants' Tables (Aggregate)... ...	O^{am}
Male Annuitants' Tables (Select)	$O^{[am]}$

FEMALE LIVES.

F instead of M, and *f* instead of *m* throughout.

Combined Summary of Data.

ALL CLASSES.

BRITISH OFFICES LIFE TABLES, 1893.

Combined Old and New Assurances. **Participating and Non-Participating.**

Symbol.	Section of Experience.	Total Number of Cards Contributed to Experience.	Number of Cases included in Experience. Select Data (after Elimination of Duplicates, &c.). Totals.	Existing.	Withdrawals.	Termi-nations.	Deaths.	Number of Years of Risk. Select Tables.	Aggregate Tables.
	MALE ASSURED LIVES.								
0^{M}	Whole-Life Assurances—Uniform Premiums...	785,222	735,079	352,271	187,037	—	195,771	9,306,357	7,659,454
0^{LM}	,, ,, Limited Premiums...	39,019	36,839	26,794	4,995	—	5,050	234,593*	410,251
0^{IM}	,, ,, Increasing Premiums	25,535	23,280	10,411	9,805	—	3,064	128,575*	207,709
0^{EM}	Endowment Assurances	144,981	140,414	97,314	30,898	6,181	6,021	947,753	897,673
0^{CM}	Contingent Survivorship Assurances	3,987	3,482	923	1,207	1,074	278	15,586*	— ‡
0^{TM}	Temporary Assurances	13,731	11,603	1,383	3,698	6,121	401	36,489*	— ‡
0^{JM}	Joint-Life Assurances	9,668	9,195	2,667	3,428	1,416	1,684	44,683*	90,171
	TOTALS—Male Assured Lives	**1,022,143**	**959,892**	**491,763**	**241,068**	**14,792**	**212,269**	**10,714,036**	**9,265,258**
	FEMALE ASSURED LIVES.								
0^{F}	Whole-Life Assurances—Uniform Premiums...	62,362	58,411	22,807	15,699	—	19,905	676,367	619,052
0^{LF}	,, ,, Limited Premiums...	799	761	487	130	—	144	8,451†	8,451†
0^{IF}	,, ,, Increasing Premiums	1,074	1,041	399	455	—	187	8,936†	8,936†
0^{EF}	Endowment Assurances	6,798	6,563	4,068	1,642	549	304	— ‡	42,646
0^{CF}	Contingent Survivorship Assurances... ...	1,552	1,436	417	537	390	92	8,446†	8,446†
0^{TF}	Temporary Assurances...	2,084	1,907	194	602	1,056	55	6,104†	6,104†
0^{JF}	Joint-Life Assurances	7,547	7,222	1,960	2,462	1,545	1,255	35,985*	77,078
	TOTALS—Female Assured Lives	**82,216**	**77,341**	**30,332**	**21,527**	**3,540**	**21,942**	**744,289**	**770,713**
	TOTALS—Male and Female Assured Lives ...	**1,104,359**	**1,037,233**	**522,095**	**262,595**	**18,332**	**234,211**	**11,458,325**	**10,035,971**
	ANNUITANTS. (BRITISH OFFICES LIFE ANNUITY TABLES, 1893).								
0^{am}	Male Annuitants	9,700	8,641	4,214	—	—	4,427	67,250	53,599
0^{af}	Female Annuitants	24,300	23,056	11,956	—	—	11,100	207,324	173,519
	TOTALS—Male and Female Annuitants ...	**34,000**	**31,697**	**16,170**	—	—	**15,527**	**274,574**	**227,118**

* First Ten years of Assurance only. † Approximate figures only, no extended Tables having been prepared. ‡ Not computed.

CONTENTS.

NOTE

AS TO THE ACCOUNT OF THE PRINCIPLES AND METHODS ADOPTED IN THE COMPILATION OF THE DATA.

In the preparation of the following text, with the illustrative Tables and Appendices, setting forth the methods followed in the Compilation and Tabulation of the data for Annuitants and for Assured Lives, and in the careful examination of the proof sheets, I have received much valued assistance from Mr. H. P. CALDERON, F.I.A., and Mr. G. GREEN, B.A. (Cantab.), A.I.A., who were engaged in the practical conduct of the Mortality Investigation from 1896 and 1897 respectively, until its completion in 1900, and who were, in July 1898, appointed by the Committee as Joint Assistant Supervisors of the work.

THOMAS G. ACKLAND,
Hon. Official Supervisor.

NOTES AS TO THE

PRINCIPLES AND METHODS ADOPTED

FOR

CLASSIFYING AND TABULATING THE DATA.

ANNUITANT EXPERIENCE.

BY

THOMAS G. ACKLAND, F.I.A.,

Hon. Official Supervisor.

ANNUITANT EXPERIENCE.

NOTES AS TO THE PRINCIPLES AND METHODS ADOPTED FOR CLASSIFYING AND TABULATING THE DATA.

I. AS TO PRELIMINARY CLASSIFICATIONS.

(1). The cards comprising the data for the Annuitant Experience entered up and supplied by the contributing Companies in accordance with the Instructions issued by the Committee (see pp. 22-24), were in the first instance examined to see, from the date of entry, whether they appeared to include, within the limits laid down by those Instructions, the whole experience of each contributing Company. The cards were next sorted according to colour—buff, Male Annuitants; blue, Female Annuitants; after which the cards of the several companies were combined. The total number of cards was (approximately) 9,700 for Male, and 24,300 for Female lives. Specimens of the form of card employed (reduced in size) are given in Appendix VII, p. 34.

II. AS TO THE METHOD ADOPTED FOR DETERMINING THE AGES AT PURCHASE.

(2). An investigation was then made as to the interval subsisting, in the case of each annuity, between the date of purchase and the preceding birthday. Appendix I gives the results, separately stated for male and female lives. It will be seen that the average interval is approximately 4·1 months in the case of Male Annuitants, and 4·3 months in the case of Female Annuitants. These results are in close agreement with those given by Mr. A. J. Finlaison, in his Report of 10th February, 1883, upon the Experience of Government Annuitants up to 1875. The proportionate numbers tabulated in Appendix I were most useful as a basis for supplementing defective data in respect of the day and month of birth or of entry.

(3). The Age at Purchase was, for purposes of tabulation, taken throughout as that attained upon the birthday nearest to the date of purchase, as determined by a comparison of the dates recorded upon the cards. The age thus deduced was that termed by Dr. Sprague (*J.I.A.* XXXI, 208) the "Commencing Age." It was found that the most practical way of arriving at the tabular age at purchase was first to modify the year of birth (by addition or deduction of 1 where necessary), so that the difference between the year of purchase and the modified year of birth would give in all cases the

nearest age at purchase. The modification of the year of birth was given effect to by marking the recorded year + or — in certain cases, determined by the rules set out in Appendix II.

(4). In cases where the dates of birth or of purchase were not fully recorded, the assumption was made that the average distribution of the interval between the dates of purchase and of previous birthday agreed substantially with that observed amongst the general body of lives, as set out in Appendix I. From the Table there given, it will be seen that amongst the Female Lives, 69 per cent. effected their contracts during the first six months of the year of age, and 31 per cent. during the second six months of the year of age, while, amongst the Male Lives, the proportions were 70 per cent. and 30 per cent. respectively. The tabular ages at purchase were then supplied, in the cases of defective data, so as to maintain the same proportionate distributions; *i.e.*, in seven cases out of ten the age last birthday, and in the other cases the age next birthday, was assumed to be the nearest age.

(5). The above percentages also afforded a basis for ascertaining the extent of the error involved in treating the Experience as if all cases were effected at their nearest age at purchase. Thus, in the tabulation of the Female Experience, 69 lives out of 100 were referred back to their last birthdays, and the remaining 31 lives referred forward to their next birthdays. This involved the assumption that the average interval between the date of purchase and the last preceding birthday was ·31 of a year, or say 3·7 months. The true average interval, as shown in Appendix I, being 4·3 months, the effect of the method adopted was to understate the age at purchase by 18 days. In a similar way, the method followed gave, in the case of Male lives, an assumed average interval of 3·6 months; and the true average interval being 4·1 months, the ages were here understated by 15 days.

(6). BAPTISMS.—In cases where the date of Baptism was supplied in lieu of the date of Birth, the cards were set aside for further consideration. These numbered about 600 for both sexes, or somewhat less than 2 per cent. of the whole number. An investigation was made of the records of a large Metropolitan parish, as to the interval between the dates of birth and of baptism, the result of which is set out in Appendix III; as, however, the data there given were hardly conclusive, the 600 cases in question were referred back to the contributing Companies for the insertion of the office age at purchase (last birthday); and the assumed nearest ages at purchase were then recorded, according

to the proportionate distribution of the cases as indicated in Appendix I.

III. AS TO THE METHODS FOLLOWED IN DETERMINING AND RECORDING THE TABULAR DURATIONS.

(7). In the case of Old Annuities, *i.e.*, those effected prior to 1st January, 1863, the durations of the individual cases *as at entry under observation* were then recorded upon the cards. The contracts being brought under observation from their anniversaries in the year 1863, the DURATION BEFORE 1863 was simply the difference between the year of purchase and 1863, such duration being necessarily integral and exact in all cases. In the case of New Annuities, *i.e.*, those effected between 1st January, 1863, and 31st December, 1892, both inclusive, the cases came under observation from the date of purchase, and no duration at entry required to be specifically recorded.

(8). The DURATION OF ANNUITY (*as at exit from observation*) was, in the case of contracts "Existing" in 1893, the difference between the year of Purchase and 1893, such duration being necessarily integral and exact, since the cases were so "Existing" on their contract-anniversaries in that year. In the case of contracts terminated by Death during the period of observation, the *curtate* duration, or number of complete years elapsed since Purchase, was recorded. A very few cases, not exceeding 100 in all, were set aside owing to defective data as to the day and month of death. In these cases, the number of years assumed as completed prior to death was determined upon the basis of a uniform distribution of the deaths over the calendar year; the following exceptional cases being, however, specially treated, in consideration of the limitations of the data:—(1) Cases of death in 1863, which necessarily followed the contract anniversary in that year; (2) cases of death in 1893, which necessarily preceded the contract anniversary in that year; (3) cases of death during the calendar year of purchase, where the date of death necessarily followed that of purchase.

(9). **Withdrawals.**—The cards, 103 in number, upon which the risk was recorded as terminating otherwise than by death, were set aside for separate examination. In 19 of the cases thus set aside, the withdrawal was closely associated, in point of date, with the purchase of another annuity upon the same life. Where the amount of the annuity was increased under the second contract, the date of such increase was treated as a point of fresh selection, precisely as if the increase had been effected by means of a further annuity contract, during the currency of which the former contract had been maintained in force. Where the amount of the annuity

was not increased by the second contract, the risk was treated as a single continuous one, the withdrawal being ignored.

(10). The remaining 84 cases of withdrawal were those involving surrender of the contract to the company, and final termination of the risk; and it was ultimately decided by the Joint Committee that these should be excluded from the Experience, but that the relative data should be separately tabulated. The Tables given on pages 204–5 of the volume of Unadjusted Annuity Data supply the number arising for each sex at each age at purchase, and the number of years of risk involved under each age, separately stated for Old, New, and Combined Annuities, and for the purposes of Select and Aggregate Tables. It will be seen that the aggregate years of risk excluded in respect of these 84 cases—without corresponding deaths—amount to 284·8, showing an average duration of about 3·4 years in each case.

IV. AS TO THE TREATMENT OF DUPLICATES.

(11). **Collocation of Duplicates.**—In order to bring together the cards relating to contracts upon the same life, the whole of the cards in respect of each sex were arranged *chronologically*, in order of date of birth, and were subsequently re-arranged *alphabetically*, in order of surname; an independent examination being made under each arrangement, for the detection of duplicates. In determining as to the identity of lives, the dates of birth and death were primarily useful, while the day and month of purchase, and even the amount of the annuity, were sometimes of assistance in deciding cases otherwise doubtful. In such cases, any two points of identity or close similarity, were, speaking generally, considered as forming ground for enquiry of the Companies, and a large number of cases were thus decided. Errors detected, as a result of these enquiries, in the data as originally supplied, were corrected in red ink upon the cards, the most fruitful sources being found to be:—(1) the entry of the date of *baptism* as that of birth; (2) the entry of *assumed* dates of death, which frequently differed in respect of contracts effected upon the same life and in the same office; the discrepancy arising from the literal observance of the instructions given in the last clause of paragraph 4 of the "Memorandum for the guidance of Companies." Thus, where two annuities were in existence upon the same life, the one payable yearly on September 1st, the other yearly on December 1st, and the last payments under the two contracts were made respectively on 1st September, and 1st December, 1885, the assumed date of death was recorded in the one case as 1st March, 1886, and in the other as 1st June, 1886.

(12). In the case of Female lives, which contributed the major portion of the Experience, the question of identity was much complicated by the numerous cases of change of name arising upon marriage or re-marriage. The cards relating to Female lives were therefore further scrutinized, and cases in which the surnames differed, but the Christian names were in close agreement, with at least one other point of identity, were specially investigated. As a result of these several courses of scrutiny and enquiry, it is believed that in all cases cards relating to the same life have been brought together.

(13). **Elimination of Duplicates.**—In accordance with the instructions of the Joint Committee, the following general principles were to be observed in the elimination of duplicates:—

SELECT TABLES—(data tabulated in respect of each Age at Purchase, and each Year of Duration):—One only of those cases, arising on the same life *at the same Age at Purchase*, to be retained.

AGGREGATE TABLES — (data tabulated in respect of Ages Attained, without regard to Age at Purchase):—One only of those cases, arising on the same life *at the same Age Attained*, to be retained.

These principles to be separately applied in the tabulation of Old Annuities, New Annuities, and Combined Annuities.

(14). As an illustration of the application of these principles, reference may be made to the Rules given in Appendix IV, and to the specimen cards given in Appendix VII. Here four contracts are supposed to be effected upon the same life, two of which were taken out prior to 1863, as Old Annuities, and two subsequently to 1862, as New Annuities. The several tabular ages at purchase, and the durations brought under observation, are here set out:—

Class of Annuity.	Tabular Age at Purchase.	Durations brought under Observation.	Period of Observation.
(*a*) Old	45	3 to 18	Age 48 to Death
(*b*) Old	48	1 to 16	„ 49 to „
(*c*) New	52	0 to 12	„ 52 to „
(*d*) New	52	0 to 12	„ 52 to „

(i) SELECT TABLES.—*Old Annuities.* In the tabulation of this Section of the Experience, the cards (*a*) and (*b*), effected at different ages at purchase, would both be included.

(ii) *New Annuities.* In this Section the card (*c*) would alone be retained, (*d*) being excluded, as effected at the same age at purchase. (*See* clause 2 (*c*) of Appendix IV.)

(iii) *Combined Annuities.* Here the cards (*a*), (*b*) and (*c*), effected at different ages at purchase, would be retained, and (*d*) would be excluded.

(iv) AGGREGATE TABLES.—*Old Annuities.* In the tabulation of this Section, the card (*a*) would alone be retained, and (*b*) would be excluded, as representing the experience of duplicate years of life, irrespective of age at purchase. (*See* clause 2 (*d*), Appendix IV).

(v) *New Annuities.* Similarly, the card (*c*) would be retained, and (*d*) would be excluded.

(vi) *Combined Annuities.* For the data to be included in this tabulation, the card (*a*) would alone be retained, and the cards (*b*), (*c*) and (*d*) would be excluded, as representing the experience of duplicate years of life, irrespective of age at purchase.

(15). It will thus be seen that the data, as regards any particular life or lives entering into the Select Tables, whether in respect of Old, New, or Combined Annuities, were not in all cases identical with those entering into the Aggregate Tables; and further that the data entering into the Combined Annuities were not in all cases represented by the sum of those entering into the separate tabulation of Old and New Annuities. It was thus necessary to provide for distinct arrangements of the data in respect of each of the tabulations (i) to (vi) specified above.

(16). It is however, evident, that, *apart from the question of duplicate contracts upon the same life*, the data for Old Annuities would be identical for Select and Aggregate Tables; and similarly with the data for New Annuities; and, further, that the combination of the sectional data would furnish the data for Combined Annuities, whether for Select or Aggregate Tables. Advantage was taken of this fact to adopt a special method of deducing the tabular data, with the object of simplifying the operations, and avoiding the necessity for repeated re-sortings, and eliminations of cards. It may be added that the method thus adopted was primarily, and perhaps exclusively, adapted for the tabulation of an experience, such as that of Life Annuitants, where the data are not complicated by the introduction, during the period of observation, of any "mode of exit" other th[illegible]*death.*

(17). The data were in the first instance divided into two main groups: the *Unduplicated* cases, consisting of lives in respect of which a single contract only entered into the experience; and the *Duplicated* cases, or those in which two or more contracts upon the same life entered into the experience. Cases in the first named group, whether Old or New Annuities, necessarily entered *in*

common into the tabulation for Select and Aggregate Tables; and their combination gave the data for Combined Annuities. In the group of duplicate contracts, however, an examination had to be made of each case, in order to determine which of the duplicates on a given life had to be retained for purposes of Select Tables; and from these again had to be selected those which entered into the tabulation of Aggregate Tables. Under each form of tabulation it was also necessary to determine which of the duplicate cases, comprised in the data for Old and New Annuities respectively, were to be included in the data for Combined Annuities.

V. AS TO THE METHODS ADOPTED FOR TABULATION OF THE DATA.

(18). **Abstracts of Data.**—The cards for tabulation were thus comprised in two divisions:—

(**A**) the main body of Unduplicated cases;

(**B**) the representative Duplicates for Select Tables. In this group were included the representative Duplicates for Aggregate Tables, constituting a sub-division (**C**). The cards included in this division and sub-division were selected, and distinctively marked, according to the rules detailed in Appendix IV.

(19). The cards included in each of these divisions were first sorted into Old and New Annuities; and, in each Section, according to tabular Age at Purchase. The cards representing *Old* Annuities were, for each age at purchase, then sorted according to Duration in 1863, and the numbers tabulated as cases "Surviving," according to age at purchase and year of duration, upon Abstracts of Data in the form shown by the specimen sheet given in Appendix V. The cards in respect of each age at purchase were then re-sorted, according to Mode of Exit, as "Dying" or "Existing;" and within each such group, according to Duration at Exit; and the numbers recorded, according to age at purchase and year of duration, upon the Abstracts of Data.

(20). The cards representing *New* Annuities in each division for each age at purchase were similarly sorted according to Mode of Exit, as "Dying" or "Existing," and, within each group, according to Duration at Exit; and the numbers recorded, according to age at purchase and year of duration, upon the Abstracts of Data.

(21). As the "New" Annuities all came under observation from the date of purchase, the number of *New* Annuities, recorded as "Entered" at duration 0, constituted the total number of cases

under observation at the particular age at purchase; and, as no "Old" Annuities came under observation at duration 0, the numbers "Entered" at that duration were identical for *New* and for *Combined* Annuities.

(22). The above processes of sorting and tabulation having been completed for each of the divisions above referred to as (**A**), (**B**) and (**C**), (the entries being made upon the corresponding lines of the Abstracts of Data under the headings Old Annuities, New Annuities, and Combined Annuities), the complete material was now available for the construction of both Select and Aggregate Tables; in the case of Select Tables by addition of the numbers recorded, at any age and year of duration, upon the lines (**A**) and (**B**); in the case of Aggregate Tables, by addition of the numbers recorded, at any age and year of duration, upon the lines (**A**) and (**C**). The following detailed explanation of the several elementary and deduced functions tabulated in the volume of Unadjusted Data will, it is hoped, render the method of tabulation perfectly clear.

(23). **SELECT TABLES.**—The elementary data, as set forth in the published volume of Unadjusted Data, on pp. 2-74 for Male, and on pp. 76-166 for Female Lives, were extracted from the records in the Abstracts of Data, entered upon lines (**A**) and (**B**), as explained above. These are tabulated in respect of each age at purchase, and each year elapsed since purchase, for Old Annuities, New Annuities, and Combined Annuities. On account of further elimination of duplicates—*See* Appendix IV, (3) (iii)—the data for Combined Annuities differed from the sum of the data as separately tabulated for Old and New Annuities in five cases, of which two were on Male Lives (both at Age at purchase 63) and three were on Female Lives (at Ages at purchase 61, 68, and 76). A footnote has been added to the Select Tables at each of these ages, calling attention to these special cases. Throughout the several columns of the Tables, dashes (—) are uniformly inserted where, from the limitations laid down in taking out the Experience, there were necessarily no data, and dots (...) are inserted where there happened to be no data at the particular age at purchase and year of duration. The numbers inserted in brackets, immediately below the sectional headings Old Annuities, New Annuities, and Combined Old and New Annuities, represent the total numbers brought under observation as Entrants in each Section, and in the Combined Sections, at the particular age at purchase.

(24). "**Entered.**" COLUMNS (2) AND (9). The numbers recorded in column (2), and repeated in column (9), opposite the figures 1 and upwards in column (1), represent the Old Annuities entering under observation at their contract anniversaries in 1863, set against the "Years elapsed since Purchase" (in column 1), which represent their true integral durations when so brought under observation. These numbers are extracted from the relative columns headed "Number Surviving" in the Abstracts of Data. The numbers recorded in column (9) of the published Tables, opposite the figure 0 in column (1), represent New Annuities, all of which come under observation from the actual date of purchase (at duration 0).

(25). "**Existing.**" COLUMNS (3), (6), AND (10). The numbers recorded in these columns represent, for the Old, New, and Combined Annuities, the cases Existing at the close of the observation, that is to say, upon their contract anniversaries in 1893, taken from the relative columns headed "Number Existing" in the Abstracts of Data. These cases are set against the "Years elapsed since Purchase" in column 1 of the published Tables corresponding to their true integral durations attained in that year. Since there can necessarily arise no "Existing" in 1893 at the same durations in the case of Old and New Annuities, the numbers set out in column (10) are throughout the same as those set out, upon the same line, in either columns (3) or (6); excepting on p. 142 (Female Lives, age at purchase 61), where, for the reason stated in § (23), the Combined experience has been reduced by elimination of duplicates.

(26). "**Died.**" COLUMNS (4), (7), AND (11). The numbers recorded in these columns represent, for the Old, New, and Combined Annuities respectively, the cases Dying during the period of observation (1863–1893), taken from the relative columns headed "Number Dying" in the Abstracts of Data, and set against the figure in column (1) of the published Tables corresponding to their *curtate* duration at death. The numbers given in column (11) represent throughout the sums of those set out in columns (4) and (7), excepting on p. 54 (Male Lives, age at purchase 63), on p. 149 (Female Lives, age at purchase 68), and on p. 157 (Female Lives, age at purchase 76), where, for the reason stated in § (23), the Combined experience has been reduced by elimination of duplicates.

(27). "**Exposed to Risk.**" COLUMNS (5), (8), AND (12). The observations in respect of cases "Entered" and "Existing"

being limited throughout by contract anniversaries, and the period of risk in respect of cases of Death being computed up to the end of the year of duration current at death, the numbers in these columns are throughout integral. The processes employed in deducing the numbers Exposed to Risk in the experience of Combined Annuities, as given in column (12) of the Select Tables on pp. 2–166, are indicated by the following formulæ:—

Let $[x]$ = the tabular age at purchase;

t = the number of years elapsed since purchase;

$\sigma_{[x]+t}$ = the cases "Entered" under observation as Survivors upon the tth anniversary following date of purchase;

$\epsilon_{[x]+t}$ = the cases "Existing", at the close of the period of observation, upon the tth anniversary following date of purchase;

$\theta_{[x]+t}$ = the cases "Died" having a curtate duration of t years since purchase, that is to say, during the currency of the $(t+1)$th year;

$E_{[x]+t}$ = the "Number Exposed to Risk" in the $(t+1)$th year following date of purchase;

Then we have $$E_{[x]+0}=\sigma_{[x]+0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

for the number exposed to risk in the year immediately following purchase;

$$E_{[x]+t}=E_{[x]+t-1}+\sigma_{[x]+t}-(\epsilon_{[x]+t}+\theta_{[x]+t-1}) \quad . \quad . \quad . \quad (2)$$

for the calculation of successive values by a continued method;

$$\text{and } E_{[x]+t}=\sum_{r=0}^{r=t}\sigma_{[x]+r}-\sum_{r=1}^{r=t}\epsilon_{[x]+r}-\sum_{r=0}^{r=t-1}\theta_{[x]+r} \quad . \quad . \quad . \quad (3)$$

for verification of intermediate or final values.

Thus, for the Combined experience of Female Lives, Age at Purchase 62 (Select Tables, p. 143), we have, employing formula (2)

$$E_{[62]+10}=E_{[62]+9}+\sigma_{[62]+10}-(\epsilon_{[62]+10}+\theta_{[62]+9})$$
$$= 469 + 6 - 28 - 21 = 426;$$

or, employing formula (3), and summing the numbers in each column between the limits indicated,

$$E_{[62]+10}=\sum_{r=0}^{r=10}\sigma_{[62]+r}-\sum_{r=1}^{r=10}\epsilon_{[62]+r}-\sum_{r=0}^{r=9}\theta_{[62]+r}$$
$$= 1{,}017 - 396 - 195 = 426.$$

(28). For Old Annuities, inasmuch as the cases come under observation from their first (or later) anniversary, and not from the

date of purchase, certain terms in the several formulæ necessarily vanish in deducing the numbers exposed to risk in column (5) of the Select Tables on pp. 2-166.

(29). For New Annuities, all values of $\sigma_{[x]+\tau}$ excepting the initial value $\sigma_{[x]+0}$ disappear; and formulæ (2) and (3) reduce to the following simple forms;—

$$E_{[x]+t} = E_{[x]+t-1} - (\epsilon_{[x]+t} + \theta_{[x]+t-1}) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$E_{[x]+t} = \sigma_{[x]+0} - \sum_{\tau=1}^{\tau=t} \epsilon_{[x]+\tau} - \sum_{\tau=0}^{\tau=t-1} \theta_{[x]+\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

(30). These several formulæ were applied to deduce the numbers exposed to risk by means of working sheets of the form set out in Appendix VI, which, with the example given above, is, it is hoped, sufficiently clear and explicit to indicate the methods followed, without further explanations.

"Unadjusted Probabilities of Dying in each of the Ten Years following Purchase."

(*a*) **Arranged according to Ages at Purchase:—**

(31). On pages 168–9 and 174–5 of the volume of Unadjusted Data are given the values of $q_{[x]+t}$ for all values of x from 20 upwards, and for all values of t from 0 to 9 inclusive, arranged so that the ages at purchase $[x]$ are the same in each horizontal row, and the durations (t) in each vertical column. The values are throughout deduced from the deaths, and the numbers exposed to risk, of the Combined Annuities, as set out in columns (11) and (12) respectively of the Select Tables on pp. 2–166.

(32). On pp. 172 and 178 of the volume are given the "Probabilities of Dying in each of the Ten years following Purchase, deduced from the data for Quinquennial groups of Ages at Purchase." These probabilities have been deduced by summing in quinary groups the deaths and the numbers exposed to risk previously employed. Thus, for the group of ages at purchase 60 to 64 (Female Lives), we have for the probability after the expiration of 4 years from purchase:—

$$q_{[60\ldots64]+4} = \frac{\theta_{[60]+4} + \theta_{[61]+4} + \theta_{[62]+4} + \theta_{[63]+4} + \theta_{[64]+4}}{E_{[60]+4} + E_{[61]+4} + E_{[62]+4} + E_{[63]+4} + E_{[64]+4}} \quad . \quad . \quad . \quad (6)$$

$$= \frac{16+20+20+21+23}{618+643+697+657+643} = \frac{100}{3,258} = .03069, \text{ as tabulated.}$$

(*β*) **Arranged according to Ages Attained:—**

(33). On pp. 170–1 and 176–7 of the volume are set out the values of $q_{[x-t]+t}$ for all values of x from 25 upwards and all values of t from

0 to 9 inclusive, arranged so that the ages attained (x) are the same in each horizontal row, and the durations (t) in each vertical column. The values are again deduced from the deaths and the numbers exposed to risk of the Combined Annuities, as set out in columns (11) and (12) respectively of the Select Tables on pp. 2–166.

(34). On pp. 173 and 179 of the volume are given the "Probabilities of Dying in each of the Ten Years following Purchase, deduced from the data for Quinquennial groups of Ages Attained." These probabilities have been deduced by summing in quinary groups the deaths and the numbers exposed to risk employed in the construction of the Tables on pp. 170–1 and 176–7. Thus, for the group of ages attained 60–64 (Female Lives), we have for the probability after the expiration of 4 years from purchase,

$$q_{[\overline{60\ldots64}-4]+4} = \frac{\theta_{[56]+4}+\theta_{[57]+4}+\theta_{[58]+4}+\theta_{[59]+4}+\theta_{[60]+4}}{E_{[56]+4}+E_{[57]+4}+E_{[58]+4}+E_{[59]+4}+E_{[60]+4}} \quad . \quad . \quad . \quad (7)$$

$$= \frac{12+9+16+9+16}{408+436+480+475+618} = \frac{62}{2,417} = \cdot 02565, \text{ as tabulated.}$$

(35). The effect of selection, so far as indicated by the unadjusted data, can be most conveniently traced along the horizontal rows in the tables on pp. 170, 171, and 173 for Male lives, and on pp. 176, 177, and 179 for Female Lives.

(36). **AGGREGATE TABLES.** — The Tables given in the published volume on pp. 182–187 for Male, and on pp. 192–197 for Female Lives, supply the data and deduced functions in respect of each age attained by the lives under observation, irrespective of their ages at purchase. The tabular results are separately stated in respect of Old Annuities, New Annuities, and Combined Annuities. Duplicates having been independently eliminated under each of these three headings, as arising at ages attained, a life is represented once only in respect of each age; and duplicate periods of risk, arising by bringing together Old and New contracts upon the same life, have been eliminated in the Combined Experience. (*See* Appendix IV.) Thus, in the Female Annuity Experience at age 50, it will be seen that in the separate tabulation of the two Sections there were 29 cases "Entered" in the Old Annuities (p. 192) and 302 in the New Annuities (p. 194). In two of these cases it was found that contracts upon the same life were represented in both the Old and New Sections, and the two duplicates were therefore eliminated from the Combined Experience.

which therefore includes 29 cases "Entered in 1863" and only 300 "Entered 1863–1893" (p. 196). The elementary data for full Aggregate Tables, as tabulated, were throughout deduced from the Abstracts of Data prepared in the form of Appendix V, by addition of the numbers recorded on lines (**A**) and (**C**); the data so deduced in respect of each age at purchase and year of duration being afterwards brought together, upon working sheets, at tabular ages attained. Thus, the number of cases tabulated as "Entered" at any age attained, were made up of the combined data for all ages at purchase, and durations, which together made up such age attained; and similarly with the cases "Dying" and "Existing."

(37). **"Age."** COLUMN (1). The age here stated is in all cases the *tabular* age attained, represented by the nearest age at purchase, increased by the number of years elapsed since purchase. For ages prior to 15, the data, which include no deaths, are grouped in smaller type at the head of the Table.

(38). **"Entered."** *OLD ANNUITIES*, COLUMN (2); *NEW ANNUITIES*, COLUMN (2); *COMBINED ANNUITIES*, COLUMNS (2) AND (3). In these columns are recorded the numbers entering upon observation at the tabular age. In the Combined Annuities (pp. 186–7 and 196–7) the cases are separately tabulated according as they are "Entered in 1863" or "Entered 1863–1893."

(39). **"Existing."** *OLD ANNUITIES*, COLUMN (3); *NEW ANNUITIES*, COLUMN (3); *COMBINED ANNUITIES*, COLUMN (4). In these columns are recorded, at the tabular age, the numbers Existing (and thus passing out of observation) on their contract anniversaries in 1893.

(40). **"Died."** *OLD ANNUITIES*, COLUMN (5); *NEW ANNUITIES*, COLUMN (5); *COMBINED ANNUITIES*, COLUMN (6). In these columns are recorded the numbers of Deaths which took place in the year following the attainment of the tabular age; thus, an annuity purchased at nearest age 40, under which the life failed in the tenth year following purchase, is tabulated as "died" at age 49.

(41). **"Exposed to Risk."** *OLD ANNUITIES*, COLUMN (4); *NEW ANNUITIES*, COLUMN (4); *COMBINED ANNUITIES*, COLUMN (5). The numbers here tabulated represent the numbers Exposed to Risk for the year following the attainment of the tabular age. Let σ_x represent the cases "Entered", and ϵ_x the cases "Existing", at age x, and θ_x the cases "Dying" in the year following completed

age x; where σ_x is of the form

$$\sum_{\tau=0}^{\tau=x} \sigma_{[x-\tau]+\tau} = \sigma_{[x]+0} + \sigma_{[x-1]+1} + \ldots\ldots + \sigma_{[1]+x-1} + \sigma_{[0]+x}$$

deduced from the Abstracts of data for Aggregate Tables; and similarly with ϵ_x and θ_x; and let E_x represent the number exposed to risk in the year following age x; then we have

$$E_x = E_{x-1} + \sigma_x - (\epsilon_x + \theta_{x-1}) \quad \ldots\ldots \quad (8)$$

for the calculation of successive values by a continued method; and

$$E_x = \sum_{a=0}^{a=x} \sigma_a - \sum_{a=0}^{a=x} \epsilon_a - \sum_{a=0}^{a=x-1} \theta_a \quad \ldots\ldots \quad (9)$$

for verification of intermediate or final values. The observations in respect of cases "Entered" and "Existing" being limited throughout by contract anniversaries, and the period of risk in respect of cases of Death being computed up to the end of the year of duration current at death, the numbers exposed to risk are throughout integral.

Thus, for the Combined Experience of Female Lives at age 60 (Aggregate Tables, p. 197) we have, employing formula (8),

$$\begin{aligned} E_{60} &= E_{59} + \sigma_{60} - (\epsilon_{60} + \theta_{59}) \\ &= 3{,}840 + 775 - 265 - 62 = 4{,}288; \end{aligned}$$

or, employing formula (9), and summing the numbers in each column, from the youngest age at which there are data to the upper limit indicated,

$$\begin{aligned} E_{60} &= \sum_{a=0}^{a=60} \sigma_a - \sum_{a=0}^{a=60} \epsilon_a - \sum_{a=0}^{a=59} \theta_a \\ &= 7{,}377 - 2{,}645 - 444 = 4{,}288. \end{aligned}$$

In the computation of the numbers exposed to risk for Aggregate Tables, working sheets were employed similar in their general plan to those adopted for Select Tables (*See* Appendix VI).

(42). **"Unadjusted Mortality Tables. Living. Dying."** *OLD ANNUITIES*, COLUMNS (6) AND (7); *NEW ANNUITIES*, COLUMNS (6) AND (7); *COMBINED ANNUITIES*, COLUMNS (7) AND (8). These Tables have been computed, by the usual methods, from the numbers exposed to risk, and the deaths, as given in the two preceding columns. As the earliest death arises, in the Female Experience, after completed age 23, and, in the Male Experience, after completed age 26, the radix of each mortality table has been conveniently taken as 100,000 living at age 20; and

the numbers living and dying at each later age have been set out to the nearest integer.

(43). **"Probabilities of Living and Dying."** *OLD ANNUITIES*, COLUMNS (8) AND (9); *NEW ANNUITIES*, COLUMNS (8) AND (9); *COMBINED ANNUITIES*, COLUMNS (9) AND (10). These functions are deduced from the numbers exposed to risk and the deaths at each age, and are stated to five places of decimals throughout.

(44). **"Complete Expectation of Life."** *OLD ANNUITIES*, COLUMN (10); *NEW ANNUITIES*, COLUMN (10); *COMBINED ANNUITIES*, COLUMN (11). The values of $\mathring{e}_x$ here set out have been deduced from the probabilities of dying as given in the preceding column. The calculations were throughout performed with the Arithmometer, for which the most convenient formula was

$$e_x = (1 + e_{x+1}) - q_x(1 + e_{x+1}) \quad . \quad . \quad . \quad . \quad . \quad (10)$$

whence the complete expectations were deduced by the customary addition of ·500. The values of $\mathring{e}_x$ are stated throughout to three places of decimals. The tabulated values of p_x, q_x, and $\mathring{e}_x$, being deduced directly from the "Exposed to Risk" and "Died," will not always agree precisely in the last place of decimals with those which would be deduced from the numbers living and dying in the Unadjusted Mortality Table, these latter functions being stated to the nearest integer.

(45). "TRUNCATED" AGGREGATE TABLES.—This expression has been employed to describe the special Tables which represent the Aggregate Experience of Combined Old and New Annuities **after excluding the experience of the first five (or the first ten) years following purchase.** As a distinguishing feature, these Tables are printed throughout in a heavier type. It was decided to include these special Tables in the volume of unadjusted data, in the hope that they might facilitate investigations as to the duration of selection, and the ultimate rate of mortality after the effect of selection may be considered to have passed off. Their publication must not, however, be understood as expressing any view as to the particular number of years (probably more than ten) during which selection is actually in effective operation in the Annuity Experience. The numbers in the several columns of the Tables set forth on pp. 188-191 for Male, and on pp. 198-201 for Female Lives, have been arrived at on the principles and methods already set forth in respect of

the full Aggregate Tables; excepting that the cases "Entered" include both those coming under observation at the expiration of the fifth (or tenth) and following years after purchase, and also those earlier entrants, surviving at the expiration of five (or ten) years from purchase; and the cases "Existing" and "Died" include only those lives whose period of exposure exceeded five (or ten) years from purchase.

VI. COMPARISON OF MORTALITY FUNCTIONS, AND ANNUITY VALUES, WITH THOSE DEDUCED FROM OTHER ANNUITY TABLES.

(46). **"Four Years' Extended Mortality Table."** RATES OF MORTALITY. For the purpose of direct comparison with the Government Annuity Table, 1883, an "Extended Mortality Table" was specially constructed for each sex, based upon the Combined Annuities, tracing the lives in Select Tables through each of the first *four* years following purchase, and then passing into a Truncated Aggregate Table, from which was excluded the experience of the first *four* years following purchase. The Tables set out on the upper portion of pages 208–9 show, for each sex, and for every fifth age attained from 50 to 80, the resulting annual rates of mortality in each of the first four years following purchase, and also the rate after the expiration of four years from purchase. The values of $q_{[x-t]+t}$, as deduced from the *unadjusted* rates of mortality, being somewhat irregular, the tabulated values have been computed from the data for quinary groups of ages, so that, for example, under the heading "Annuity Experience (1863–1893)," the rate of mortality at "central age 60" (Female Lives) in the second year following purchase, as set out on page 209, is deduced by the formula:—

$$q_{\overline{[58\ldots62]-1}+1} = \frac{\theta_{[57]+1}+\theta_{[58]+1}+\theta_{[59]+1}+\theta_{[60]+1}+\theta_{[61]+1}}{E_{[57]+1}+E_{[58]+1}+E_{[59]+1}+E_{[60]+1}+E_{[61]+1}} \quad \ldots \quad (11)$$

$$= \frac{4+5+7+13+16}{529+594+604+751+790} = \frac{45}{3,268} = \cdot 01377.$$

The rates of mortality taken from Mr. A. J. Finlaison's Report dated 10th February, 1883, and appended for comparison, are the adjusted values at the ages given.

(47). EXPECTATIONS OF LIFE.—The curtate Expectations of Life have also been specially computed, for each sex, from the Extended Mortality Table above referred to, and are set out on the lower portion of pages 208-9. The values given

are those for every fifth age attained at the date of purchase, and at the end of each of the first four years following purchase. Thus, for Female Lives, at age attained 65, duration 2, the value 12·77, is that of $e_{[63]+2}$, the expectation at age 65 attained, in respect of a life aged 63 at purchase. It will be observed that whilst, for Male Lives, the curtate expectations of life as here tabulated do not upon the whole differ very materially from those brought out by the Government Annuity Tables, 1883, there appears a tendency for the values brought out by the new experience slightly to exceed those deduced from the Government Tables. For Female Lives it will be seen that the values of the expectations of life by the new experience decidedly exceed those brought out by the Government Tables, there being an excess at every age and duration here tabulated, excepting only $e_{[75]+0}$ and $e_{[79]+1}$. Upon the average the Female expectation of life by the new Table appears to exceed that deduced from the Government Annuity Tables, 1883, by fully half a year.

(48). **Aggregate Tables.** The Tables given on pages 210–1 of the published volume show, for each sex, and for ages attained of 50 and upwards, the annual rates of mortality, deduced (1) from the Combined Annuities; (2) from the several investigations of the Government Annuity Experience dated 1883, 1860, and 1829; (3) from the French Annuity Experience ("R F", *J.I.A.*, XXXIII, 485). There are further included rates deduced (4) from the H^{M} Table, and (5) from the Carlisle Table. As the unadjusted rates based upon the new experience show somewhat irregular results, the rates as graduated by Mr. Woolhouse's formula are also supplied. The corresponding curtate expectations of life, as deduced from each of the above-named Tables, are given on pages 212–3 for every fifth age attained. It will be observed that the adjusted and unadjusted expectations as deduced from the new experience, are practically identical; also that for Male Lives the expectations by the new experience are generally higher than those brought out by the Government Annuity Tables, or the French Annuity Experience; whilst, for Female Lives, the expectations by the new experience are in most instances much higher than those of the Government Annuity Experience 1860 and 1883, or the French Annuity Experience; also that the Government Annuity Tables of 1829, for Female Lives, show at most ages expectations even greater than those of the new experience.

(49). **Select Annuity Values.** By way of further comparison, the Tables on pages 214–5 show, for each sex, and at every fifth age,

the value at date of purchase of an annuity, at 3 per cent., by the Government Annuitant Experience, 1883, as given in Mr. A. J. Finlaison's Report (pp. 32–3); also that approximately deduced from the Annuitant Experience 1863–1893 (Combined Annuities,—Unadjusted Data). In order to obtain greater smoothness in the latter, they were computed by summing the data for quinary groups of ages at purchase. Thus, for example, the annuity value at "central age 60" is given by the formula

$$a_{[58\,..\,62]}=\Sigma v^t\,{}_tp_{[58\,...\,62]}$$

where ${}_tp_{[58\,...\,62]}$ is the continued product of the function

$$1-\frac{\theta_{[58]+n}+\theta_{[59]+n}+\theta_{[60]+n}+\theta_{[61]+n}+\theta_{[62]+n}}{E_{[58]+n}+E_{[59]+n}+E_{[60]+n}+E_{[61]+n}+E_{[62]+n}},$$

for values of n ranging from 0 to $(t-1)$. The formula adopted for deducing the annuity value at date of purchase was

$$\begin{aligned}a_{[58\,...\,62]}&=vp_{[58\,...\,62]}(1+a_{[58\,...\,62]+1})\\&=vp_{[58\,...\,62]}\{1+vp_{[58\,...\,62]+1}(1+a_{[58\,...\,62]+2})\}=\text{etc., etc.}\end{aligned}$$

This method, it will be seen, required the calculation of all successive values of the annuity $a_{[58\,...\,62]+t}$ for the determination of the annuity value at purchase $a_{[58\,...\,62]}$, but it had the advantage of being suitable for working with the arithmometer. The values of p were throughout computed from the data for Select Tables, as is indicated by the above formulæ.

(50). Inasmuch as the mean age of the entrants at the ages at purchase 58, 59, 60, 61, and 62, is not 60 exactly, but a modified age, say 60′, the value of $a_{[58\,...\,62]}$ thus obtained (see column 3, p. 214), does not correctly represent $a'_{[60]}$, the value of the annuity at the central age 60, but rather that of $a'_{[60']}$, an annuity at the mean age. To obtain the corrected value, the following formula, based upon Central First Differences, was employed:—

$$a'_{[x]}=a'_{[x']}+\frac{x-x'}{(x+5)'-(x-5)'}\left\{a'_{[(x+5)']}-a'_{[(x-5)']}\right\},$$

where x' represents the mean age of the total entrants at the several ages at purchase $x-2$, $x-1$, x, $x+1$, and $x+2$; whilst $(x+5)'$ represents the mean age of the entrants at ages $x+3$, $x+4$, $x+5$, $x+6$, and $x+7$; and similarly for $(x-5)'$. The calculation of the several values of the mean age x' given in column (2), was made by means of the formula

$$x'=x+\frac{2(E_{[x+2]+0}-E_{[x-2]+0})+(E_{[x+1]+0}-E_{[x-1]+0})}{E_{[x-2]+0}+E_{[x-1]+0}+E_{[x]+0}+E_{[x+1]+0}+E_{[x+2]+0}};$$

Thus, for example, at central age 60, for Male lives,

$$x' = 60 + \frac{2(277-193)+(275-192)}{193+192+278+275+277} = 60{\cdot}21.$$

and $$a'_{[60]} = a'_{[60{\cdot}21]} + \frac{60-60{\cdot}21}{65{\cdot}04-55{\cdot}11}\left\{a'_{[65{\cdot}04]} - a'_{[55{\cdot}11]}\right\}$$

$$= 10{\cdot}785 + \frac{60-60{\cdot}21}{65{\cdot}04-55{\cdot}11}\left\{9{\cdot}015 - 12{\cdot}633\right\} = 10{\cdot}862,$$

as tabulated for Male lives in column (4) on p. 214.

(51). The above approximate method of deducing some indications of the annuity values at date of purchase from the Unadjusted Data, was tentatively adopted, pending the investigation and publication of a scientifically based graduation. The graduation having since been completed, it may be convenient to append for comparison the graduated annuity values as at date of purchase, computed at 3 per cent. at every fifth age from 40 to 80 inclusive, for Male and Female Lives. These are as follow:—

Age at Purchase. [x]	Graduated Value of $a_{[x]}$ at 3 per cent.	
	Males.	Females.
40	17·603	18·257
45	16·061	16·930
50	14·403	15·514
55	12·661	13·964
60	10·881	12·230
65	9·121	10·333
70	7·441	8·406
75	5·898	6·614
80	4·537	5·054

On comparing these graduated values with those approximately deduced, as given in column (4) on pages 214 and 215 of the volume of Unadjusted Data, it will be observed that the differences are not very considerable, excepting at ages 40, 45 and 50 for Male Lives, where the original data were very scanty; the Entrants ("New" Annuities) at the grouped ages at purchase 38–42, 43–47 and 48–52, being for Male Lives, 134, 239 and 521 respectively.

THOMAS G. ACKLAND,

Hon. Official Supervisor.

COPY OF SPECIAL INSTRUCTIONS ISSUED TO COMPANIES CONTRIBUTING TO THE ANNUITANT EXPERIENCE.

NEW COLLECTIVE MORTALITY EXPERIENCE

IN PREPARATION BY THE INSTITUTE OF ACTUARIES AND THE FACULTY OF ACTUARIES.

MEMORANDUM

FOR THE GUIDANCE OF THE COMPANIES IN FILLING IN THE MORTALITY EXPERIENCE CARDS.

ANNUITY NOMINEES.

1. IT is intended that the New Collective Mortality Experience shall include (*a*) annuities existing on the books of the Companies on the anniversaries in 1863 of the dates of entry, and (*b*) annuities granted between 1st January 1863 and the 31st December 1892. A form of card will be supplied for each of these separate classes, and specimens are enclosed herewith. That headed "Old Annuities" is intended for class (*a*) above named, and that headed "New Annuities" for class (*b*). A card should be written for each annuity that comes within the limits of the experience.

2. The male lives are to be distinguished from the female; and for the male lives buff cards are to be used, and for the female, blue cards.

3. Only those annuities are to be included which are on lives resident in the United Kingdom at the date of entry. Thus, cards are not to be written for annuities granted through agencies abroad, or for annuities granted in the United Kingdom on lives residing abroad at the date of entry.

4. Only those annuities which at the date of the contract were immediate, and for the whole of life, should be included. Thus, cards should not be written for deferred annuities, temporary annuities, contingent survivorship annuities, or annuities on joint lives; but annuities on joint lives and the survivor should be included, and cards should be written for each life involved. In all cases of annuities on joint lives and the survivor, the letter "S" (signifying survivor) should be placed under the heading "Remarks."

5. Only those annuities should be included which were granted at tabular rates for the ages at entry. Annuities set up in connection with reversionary transactions of any kind should be excluded, only those annuities being included which were granted to the public in consideration of cash payments.

6. Only direct annuities of the Company are to be included; and therefore cards are not to be written for annuities granted to other Offices.

7. It will be noticed that two descriptions of type appear on the cards. It is intended that the Companies shall fill in the particulars required only under the large capital type, and that those asked for in the small Roman type shall be filled in by the Institute and the Faculty of Actuaries.

8. Taking in order the lines in capital type upon the cards, the following explanations may be useful.

9. **No.**..... The Contract No. should be inserted here.

10. **£**..... The amount here required is the sum per annum to the nearest £ payable under the annuity contract.

11. **"Life."** It is desirable that the full Surname and the first Christian name, and the initials of other Christian names of the Life should be given.

The Surname should be placed on the first line, and Christian name or names on the second. In the case of a compound Surname, such as John Brown-Smith, the last name only should be treated as Surname, the remainder being treated as part of the Christian name and given in full after the Christian name on the second line: thus { Smith, / John Brown.

Similarly in the case of a Surname with such a prefix as "de", "von", "van", "van der", &c., *e.g.*, "Van Tromp", only the Surname itself, *e.g.*, "Tromp", should be placed on the first line, and the prefix "Van", &c., should be placed on the second line after the Christian names. If, however, the prefix is actually incorporated in the name, *e.g.*, Vanderbilt, then the whole should appear on the first line as Surname.

In the case of a female who has changed her name by marriage, the name under which she was nominated for the annuity should be given on these two lines, and her maiden name, or her married name, as the case may be, should, if possible, be given under "Remarks."

In the case of a peer, the family name and the Christian names should be given on these two lines, and the title should be given under "Remarks."

12. **"Date of Birth."** The date of birth should be given with as much accuracy as possible. The day of the month should be inserted under the letter "D" in the column before the hyphen, and the number of the month, thus 7 for July, should be inserted after

the hyphen, the year being given in the ruled column on the right, under the word "year." If the exact date of birth cannot be given, such particulars as are possible should be supplied in this line.

13. **"Date of Entry."** Here should be written the date when the annuity commenced to run.

14. **"Date of Exit."** In the great majority of cases exit will have been caused by death, and the date of death should be entered here. Should the annuity have been cancelled by surrender, the date of surrender should be given. Should the month of death only be known this should be inserted. When even the month of death is unknown, the date of death is to be assumed to be the day half-way between the last due-date of payment and the next day on which the annuity would have been payable if the annuitant had not died.

15. **"Mode of Exit."** The letter "D", to be placed within the brackets, will mean that the exit was caused by death, and the letter "T" by termination in any other way.

16. The observations are to close with the anniversary of the contract in 1893. If the contract still remained in force at its anniversary in 1893, the lines Date of Exit and Mode of Exit should be left blank; and it will be assumed in all cases where no mark is made on them, that the contract was still running at the close of the observations.

17. For the sake of distinguishing the different Companies, so that the cards may be returned after they have been used for the Mortality Experience, each Company shall have a distinguishing number or letter, to be approved by the Institute and the Faculty of Actuaries, printed at the foot of the card. The Company for its own purposes may make such remarks on the back of the card as may be thought desirable; but it is particularly requested that no marks except those above mentioned be made on the face of the card.

18. The writing and figures on the cards should be made as distinct as possible, and the figures should be ranged under each other so that there may be no difficulty in reading them; and in the cards for old annuities, under the heading "year", the figures should be ranged above and below those for 1863 which are printed, so that in subsequently dealing with the cards there may be no difficulty in making additions or subtractions.

19. The cards of each Company, when all completed, should be sent in to the Institute of Actuaries or to the Faculty of Actuaries arranged in any order that the Company may find convenient.

20. If any further explanations be required they will be supplied on application to the Honorary Secretaries of the Institute of Actuaries, Staple Inn Hall, Holborn, London, W.C., or to the Honorary Secretary of the Faculty of Actuaries, Edinburgh.

August 1, 1894.

Appendix I.

ANNUITANT EXPERIENCE 1863-1893

Table showing the proportionate Number of Contracts effected by Male and Female Annuitants, scheduled according to the Interval (in months) between the Date of Purchase and the preceding Birthday.

Interval (in Months) between Date of Purchase and preceding Birthday. (1)	Proportionate Number of Contracts (per 10,000) Effected by					
	Male Annuitants (2)		Female Annuitants (3)		Male and Female Annuitants (4)	
0 to 1	2,149		2,018		2,054	
1 to 2	1,402		1,322		1,345	
2 to 3	1,047		923		956	
3 to 4	916		928		925	
4 to 5	787		884		858	
5 to 6	716		809		784	
	—	7,017	—	6,884	—	6,922
6 to 7	747		764		759	
7 to 8	669		729		713	
8 to 9	575		581		579	
9 to 10	446		458		454	
10 to 11	333		321		324	
11 to 12	213		263		249	
	—	2,983	—	3,116	—	3,078
Totals		10,000		10,000		10,000
Average Interval	4·1 Months		4·3 Months		...	

Mean Age Method:—

Assumed Interval	6·0 Months	6·0 Months
Deviation ...	1·9 Months (57 days)	1·7 Months (51 days)

Nearest Age Method:—

Average Interval	3·6 Months	3·7 Months
Deviation ...	0·5 Months (15 days)	0·6 Months (18 days)

Appendix II.

RULES FOR OBTAINING THE NEAREST AGE AT PURCHASE BY MODIFICATION OF THE YEAR OF BIRTH.

(1) Modification to be applied to the Year of Birth recorded on the cards in the following cases:—

(*a*) Where Day and Month of Birth *precede* Day and Month of Entry by *more than six months*, mark the Year of Birth (−)

(*b*) Where Day and Month of Birth *follow* Day and Month of Entry by *more than six months*, mark the Year of Birth (+)

(*c*) Where the interval between Day and Month of Birth and Day and Month of Entry is *exactly six months* :—

(i) If the Day and Month of Birth *precede* the Day and Month of Entry, mark the Year of Birth, in *one-half* of the cases, (−)

(ii) If the Day and Month of Birth *follow* the Day and Month of Entry, mark the Year of Birth, in *one-half* of the cases, (+)

(2) In all cases to obtain the Nearest Age at Purchase deduct the Year of Birth (modified ±1 as marked in the above cases) from the Year of Entry.

Appendix III.

BAPTISMS REGISTERED IN THE PARISH OF ST. JOHN OF WAPPING, 1760–1870.

One hundred and twenty cases were extracted promiscuously from the entries in the several months, January, 1760; February, 1770; March, 1780; and so on, up to December, 1870;—ten from each month :—

Interval between Birth and Baptism.	Number of Cases.	Interval between Birth and Baptism.	Number of Cases.
0– 9 days	6	Brought forward	97
10–19 „	12		
20–29 „	36		
	—54		
1– 2 months	31	1– 2 years	8
2– 3 „	5	2– 3 „	4
	—36		
3– 4 „	2	3– 4 „	1
4– 5 „	2	4– 5 „	5
5– 6 „	1	5– 6 „	1
6– 7 „	0	6– 7 „	2
7– 8 „	1	7– 8 „	0
8– 9 „	1	8– 9 „	1
9–10 „	0	9–10 „	0
10–11 „	0	10–11 „	0
11–12 „	0	11–12 „	1
	— 7		—23
Carried forward	97	Total	120

Appendix IV.

NOTES AS TO THE SELECTION AND DISTINCTIVE MARKING OF DUPLICATES, AND AS TO THE COMPOSITION OF DATA FOR SELECT AND AGGREGATE TABLES.

SELECTION, AND DISTINCTIVE MARKING, OF DUPLICATES.

(1) The symbols employed for distinctively marking, by india-rubber stamps, the cards representing duplicates upon the same life, and the cases to which they severally applied, were as follows:—

"X." Cases whose periods of exposure were duplicated by those of other cases effected upon the same life and at the same *Age at Purchase*, and which were therefore excluded from both *Select* and *Aggregate* Tables.

"C S." The remaining cases, selected as representative duplicates for *Combined* Annuities, when tabulated in the form of *Select* Tables. The cases so marked, when sorted, according to the printed indication upon the cards, as "Old Annuities" and "New Annuities," were also the representative duplicates for the separate tabulation of Old and New Annuities. (It was thus unnecessary to have a distinctive symbol "S").

"C A." Cases (already marked "C S" as above) of Old or New Annuities, which showed, in respect of each life, the longest continuous exposure at ages passed through, in the experience of Old or New Annuities, and also of Combined Annuities; and which therefore entered into the *Aggregate* Tables for the Old or New Section, and for the *Combined* Sections.

"A." Cases of New Annuities (already marked "C S" as above) where an Old Annuity upon the same life had already been marked "C A." The selected New Annuities were those which showed the longest continuous exposure, in respect of each life, at ages passed through; and the cards thus marked entered into the *Aggregate* Tables for *New* Annuities only, but not for Combined Annuities.

RULES FOR MARKING THE DISTINCTIVE SYMBOLS UPON THE CARDS.

(2) The following Rules, which apply throughout to cases of two or more contracts effected upon the same life, specify the processes followed for the selection and marking of the cards:—

Preliminary.—Sort out the *Withdrawals*, and select therefrom those cases where the termination of the older contract is associated, in point of date, with the purchase of another contract effected upon the same Life. In cases where the later contract increases the amount of the Annuity, write a fresh card, combining the exposures of the two contracts; mark the Withdrawal card "X", and leave the card representing the later contract unmarked. In cases where the later contract does not increase the amount of the Annuity, write a fresh card, combining the exposures of the two contracts, and mark both the constituent cards "X." Mark all remaining Withdrawal cases "X."

Cards marked "X" are excluded from the experience, and from the operation of the following Rules.

(*a*) Sort the cards (including any re-written cards) upon the same life in order according to tabular age at purchase.

(*b*) Where there is one card only at any age at purchase, mark the card "C S."

(*c*) Where there are two or more cards at the same age at purchase, select the case in which the tabular age at purchase most closely approximates to the true age at purchase; mark that card "C S", and the remaining cards "X." (*See* also note (iii) on next page.)

(*d*) From the cards upon the same life, marked "C S," select the one at the earliest age at purchase, marking the same "C A."

(*e*) Where the selected card marked "C A" is an Old Annuity, examine any New Annuities (upon the same life) separately, and select the card which would have been marked "C A" if the Old Annuities had been absent, and mark such selected card "A."

(3) A few special and exceptional cases, arising in the process of thus selecting and marking the cards, may be here referred to:—

(i) *Annuity contracts effected at different dates, upon the same life, of which one (or more) was Existing at the close of the period of observation, and one (or more) was terminated by Death in the year* 1893. As the period of observation of each contract did not extend beyond its anniversary in 1893, such a case arose whenever

the death of an Annuitant happened, in 1893, between the anniversaries in that year of the different contracts upon the life. Where two or more of such contracts were effected at the same nearest age at purchase (but at different calendar dates), a choice had to be made for the purpose of Select Tables; where the several contracts were effected at different ages at purchase, they were all included in the Select Tables, but a choice of one had to be made for the Aggregate Tables. In the former case, the application of the above rules involved the selection of that contract in which the nearest age at purchase most closely approximated to the true age; and in the latter case, of the contract effected at the earliest age at purchase. In consequence of this selection in certain cases of the "Existing" contract, there were excluded from the Select Tables 5 deaths (4 on Male, 1 on Female, lives); and from the Aggregate Tables, 17 deaths (5 on Male, 12 on Female, lives).

(ii) *Contracts effected upon the same life and at the same tabular age at purchase.* The following special cases were dealt with, under rule (*c*), by selection of the contract in which the tabular age at purchase most closely approximated to the true age:—

(*a*) Two or more cases "Existing" in 1893, the integral durations differing by one year.

(*b*) Two or more cases of "Death" during the period of observation, the curtate durations differing by one year.

(iii) *Contracts effected upon the same life, and at the same tabular age at purchase, one (in* 1862*) as an Old Annuity, the other (in* 1863*) as a New Annuity.* Here one case would be included in the data for Select Tables, Old Annuities, and the other for New Annuities. In the tabulation of Combined Annuities, the case selected (marked "C S") was that in which the tabular age at purchase most closely accorded with the true age; and the other case was exceptionally marked "[C]S," and eliminated as a duplicate. Only five cases of this type actually arose in the Annuity Experience, two on Male and three on Female lives. In each of the cases the card retained for the tabulation of Combined Annuities, and marked "C S" happened to be the Old Annuity upon the life.

COMPOSITION OF DATA FOR SELECT AND AGGREGATE TABLES.

GROUP (A). Unduplicated cases, being those in which a single contract only upon each life had been effected. These cases, which bore no distinctive markings, other than the words "Old Annuities" or "New Annuities" printed at the top of the cards, entered into the tabulation of both Select and Aggregate Tables for Old or New, and also for Combined, Annuities.

GROUP (B). Representative duplicates entering into the construction of *Select* Tables; comprising Old and New Annuities distinctively marked

"CS," or "CS. CA"; and New Annuities marked "CS. A." (The marking "CS" being thus common to all). The data for Combined Annuities included all cards thus marked, saving the exceptional cases referred to under clause (iii) above, and marked "[C]S."

GROUP (C). Representative duplicates entering into the constuction of *Aggregate* Tables; comprising the following, selected from Group (B):— Old and New Annuities marked "CS. CA"; and New Annuities marked "CS. A." (The marking "A" being thus common to all). The data for Combined Annuities included the cards marked "CS. CA" only.

The sectional data in groups (A), (B) and (C) being then entered on the corresponding lines of the abstracts of data, the addition of the tabulated data on lines (A) and (B) forms the complete data for Select Tables; whilst the addition of lines (A) and (C) forms the complete data for Aggregate Tables.

The following scheme shows graphically the manner in which the cards in the several groups entered into the construction of Select and Aggregate Tables:—

SECTION.	Representative Duplicates for Select Tables.	Unduplicated Cases.	Representative Duplicates for Aggregate Tables.
	Group (B)	Group (A)	Group (C)
	Distinctive Markings of the Cards.		
"OLD" Annuities	CS "OLD"	"OLD"	CA "OLD"
"NEW" Annuities	CS "NEW"	"NEW"	A "NEW", CA "NEW"
Combined "OLD" and "NEW" Annuities	CS "OLD", CS "NEW"	"OLD" "NEW"	CA "OLD", CA "NEW"
	Group (B)	Group (A)	Group (C)

Select Tables.

Aggregate Tables.

SPECIMEN CARDS, illustrating the original entries, the records of Ages and Durations (which were made by means of india-rubber stamps), and the distinctive markings of Duplicates, are given in Appendix VII. The particulars as to "Age in 1863" and "Age at Exit" (which had been provided for, in view of a possible tabulation by ages passed through instead of durations) were not ascertained, or recorded upon the cards.

ANNUITANT EXPERIENCE 1863-1893

AGE AT ENTRY‡ 62. **ABSTRACT OF DATA** FEMALE LIVES

YEARS OF DURATION *	AGES PASSED THROUGH		"OLD" ANNUITIES (Contracts effected prior to 1863, and subsisting on their Policy-Anniversaries in that year) Number brought under Observation } = Number SURVIVING† on Policy-Anniversaries in 1863	Number DYING between 1863 and 1893	Number EXISTING on Policy-Anniversaries in 1893	"NEW" ANNUITIES (Contracts effected between 1st January 1863, and 31st December 1892) Number brought under Observation } = Number DYING between 1863 and 1893	Number EXISTING on Policy-Anniversaries in 1893	COMBINED "OLD" AND "NEW" ANNUITIES (Contracts subsisting in 1863, and those effected between 1st January 1863, and 31st December 1892) Number brought under Observation } = Number SURVIVING† on Policy-Anniversaries in 1863	Number DYING between 1863 and 1893	Number EXISTING on Policy-Anniversaries in 1893	AGES PASSED THROUGH		YEARS OF DURATION *
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)		(12)
0	62	B	...	...	...	2	...	...	2	...	B		
		A	...	...	...	8	...	...	8	...	A	62	0
		C	...	...	...	...	...	...	...	...	C		
1	63	B	4	...	...	2	16	4	2	16	B		
		A	15	1	...	10	30	15	11	30	A	63	1
		C	...	...	...	...	3	...	...	3	C		
2	64	B	5	...	...	...	17	5	...	17	B		
		A	7	1	...	9	29	7	10	29	A	64	2
		C	1	...	...	...	7	1	...	7	C		
3	65	B	7	...	...	6	19	7	6	19	B		
		A	13	...	...	16	47	13	16	47	A	65	3
		C	4	...	...	2	8	4	2	8	C		
4	66	B	5	...	...	5	13	5	5	13	B		
		A	8	...	...	15	27	8	15	27	A	66	4
		C	4	...	...	1	4	4	1	4	C		
5	67	B	5	1	...	7	12	5	8	12	B		
		A	5	1	...	13	30	5	14	30	A	67	5
		C	5	...	...	5	3	5	5	3	C		
6	68	B	3	...	...	4	11	3	4	11	B		
		A	8	2	...	11	17	8	13	17	A	68	6
		C	2	...	...	3	4	2	2	4	C		
7	69	B	4	1	...	7	24	4	8	24	B		
		A	7	5	...	21	20	7	26	20	A	69	7
		C	4	...	...	5	16	4	4	16	C		
8	70	B	5	2	...	6	14	5	8	14	B		
		A	11	1	...	17	20	11	18	20	A	70	8
		C	3	1	...	4	6	3	4	6	C		
9	71	B	2	1	...	3	6	2	4	6	B		
		A	11	3	...	14	16	11	17	16	A	71	9
		C	...	1	...	2	5	...	3	5	C		
Totals carried forward	.....	B	40	5	...	42	132	40	47	132	B	.....	Totals carried forward
	.....	A	85	14	...	134	236	85	148	236	A	.....	
	.....	C	23	2	...	22	56	23	21	56	C	.....	
Contribution to *Select* Tables: Contribution to *Aggregate* Tables:			Old Annuities (A+B) (A+C)			New Annuities (A+B) (A+C)		Combined Annuities (A+B) (A+C)			Contribution to *Select* Tables: Contribution to *Aggregate* Tables.		

NOTE.—The headings ultimately adopted by the Joint Committee for the published Tables were as under:—
* "Years elapsed since Purchase" (columns 1, 12). † "Entered" (columns 3, 8). ‡ "Age at Purchase" (heading).

SELECT TABLES (A+B) **WORKING SHEET FOR EXPOSED**

AGE AT ENTRY 62=[x]

YEARS OF DURATION	AGES PASSED THROUGH	"OLD" ANNUITIES (Contracts effected prior to 1863, and subsisting on their Policy-Anniversaries in that year) NUMBER UNDER OBSERVATION (SURVIVORS)=168							"NEW" (Contracts effected and 31st ENTRANTS ($\sigma_{[x]}$)	
		Survivors in 1863	Deaths 1863–1893	Existing in 1893	Total Decrement	NET MOVEMENT		Exposed to Risk	Deaths 1863–1893	Existing in 1893
(t)	$[x]+t$	$\sigma_{[x]+t}$	$\theta_{[x]+t-1}$	$\epsilon_{[x]+t}$	$(\theta+\epsilon)$	(G) $=\sigma_{[x]+t}-(\theta+\epsilon)$		$\Sigma^t(G)$ $=E_{[x]+t}$	$\theta_{[x]+t-1}$	$\epsilon_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
0	62	...	...	...	...	+ ...	− ...	...	...	...
1	...	19	...	...	...	19	...	19	10	46
2	...	12	1	...	1	11	...	30	12	46
3	...	20	1	...	1	19	...	49	9	66
4	...	13	...	...	...	13	...	62	22	40
5	...	10	...	...	...	10	...	72	20	42
6	...	11	2	...	2	9	...	81	20	28
7	...	11	2	...	2	9	...	90	15	44
8	...	16	6	...	6	10	...	100	28	34
9	...	13	3	...	3	10	...	110	23	22
10	72	6	4	...	4	2	...	112	17	28
11	...	6	7	...	7	...	1	111	17	28
12	...	2	8	...	8	...	6	105	21	21
13	...	4	6	...	6	...	2	103	15	13
14	...	5	5	...	5	...	...	103	16	16
15	...	3	12	...	12	...	9	94	12	13
16	...	1	12	...	12	...	11	83	6	8
17	...	3	13	...	13	...	10	73	12	15
18	...	2	8	...	8	...	6	67	8	7
19	...	2	7	...	7	...	5	62	9	5
20	82	2	5	...	5	...	3	59	15	2
21	...	2	9	...	9	...	7	52	5	8
22	...	1	4	...	4	...	3	49	5	3
23	...	1	5	...	5	...	4	45	7	7
24	...	1	8	...	8	...	7	38	3	3
25	...	...	8	...	8	...	8	30	2	...
26	...	1	9	...	9	...	5	22	...	6
27	...	1	4	...	4	...	3	19	5	...
28	...	...	8	...	8	...	8	11	...	1
29	...	...	2	...	2	...	2	9	...	...
30	92	...	5	...	5	...	5	4	...	...
31	...	...	1	...	1	...	1	3	...	...
32	...	...	1	...	1	...	1	2	...	...
33	...	...	1	...	1	...	1	1	...	...
34	...	...	...	...	...	...	...	1	...	...
35	...	...	1	...	1	...	1	...	...	...
36	...	...	...	...	...	...	...	...	...	...
37	...	...	...	...	...	...	...	...	...	...
38	...	...	...	...	...	...	...	...	...	...
39	...	...	...	...	...	...	...	...	...	...
	...	168	168	...	112	112	112	1,871	334	552

* When $t=0$, $\sigma_{[x]+t}=$ and for all other values

1893

DEDUCING NUMBERS **FEMALE LIVES**

TO RISK **AGE AT ENTRY 62=[x]**

ANNUITIES between 1st January 1863 December 1892) =886		COMBINED "OLD" AND "NEW" ANNUITIES (Contracts subsisting in 1863 and those effected between 1st January 1863 and 31st December 1892 NUMBER UNDER OBSERVATION {SURVIVORS=168 / ENTRANTS =886} =1054							AGES PASSED THROUGH	YEARS OF DURATION
Net Movement	Exposed to Risk	Survivors in 1863	Deaths 1863-1893	Existing in 1893	Total Decrement	NET MOVEMENT		Exposed to Risk		
(G) $=\sigma_{[x]+t}-(\theta+\epsilon)$	$\Sigma^t(G)=E_{[x]+t}$	$\sigma_{[x]+t}$	$\theta_{[x]+t-1}$	$\epsilon_{[x]+t}$	$(\theta+\epsilon)$	(G) $=\sigma_{[x]+t}-(\theta+\epsilon)$		$\Sigma^t(G)=E_{[x]+t}$	$[x]+t$	(t)
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
+886	886	...	...	...	...	+886	− ...	886	62	0
− 56	830	19	10	46	56	...	37	849	...	1
58	772	12	13	46	59	...	47	802	...	2
75	697	20	10	66	76	...	56	746	...	3
62	635	13	22	40	62	...	49	697	...	4
62	573	10	20	42	62	...	52	645	...	5
48	525	11	22	28	50	...	39	606	...	6
59	466	11	17	44	61	...	50	556	...	7
62	404	16	34	34	68	...	52	504	...	8
45	359	13	26	22	48	...	35	469	...	9
45	314	6	21	28	49	...	43	426	72	10
45	269	6	24	28	52	...	46	380	...	11
42	227	2	29	21	50	...	48	332	...	12
28	199	4	21	13	34	...	30	302	...	13
32	167	5	21	16	37	...	32	270	...	14
25	142	3	24	13	37	...	34	236	...	15
14	128	1	18	8	26	...	25	211	...	16
27	101	3	25	15	40	...	37	174	...	17
15	86	2	16	7	23	...	21	153	...	18
14	72	2	16	5	21	...	19	134	...	19
17	55	2	20	2	22	...	20	114	82	20
13	42	2	14	8	22	...	20	94	...	21
8	34	1	9	3	12	...	11	83	...	22
14	20	1	12	7	19	...	18	65	...	23
6	14	1	11	3	14	...	13	52	...	24
2	12	...	10	...	10	...	10	42	...	25
6	6	1	9	6	15	...	14	28	...	26
5	1	1	9	...	9	...	8	20	...	27
1	...	...	8	1	9	...	9	11	...	28
...	...	...	2	...	2	...	2	9	...	29
...	...	...	5	...	5	...	5	4	92	30
...	...	...	1	...	1	...	1	3	...	31
...	...	...	1	...	1	...	1	2	...	32
...	...	...	1	...	1	...	1	1	...	33
...	...	...	...	...	...	...	...	1	...	34
...	...	...	1	...	1	...	1	...	...	35
...	...	...	...	...	...	...	...	...	...	36
...	...	...	...	...	...	...	...	...	...	37
...	...	...	...	...	...	...	...	...	...	38
...	...	...	...	...	...	...	...	...	...	39
...	8,036	168	502	552	1,054	886	886	9,907	...	...

the number of Entrants,
of t, $\sigma_{[x]+t}=0$.

Appendix VII.

SPECIMEN CARDS (two-thirds of full size).

(a)

CS OLD ANNUITIES. CA

NO. 1421 £ 105

M. LIFE { Marner, Silas.

DATE—	D. M.	YEAR.	
OF BIRTH	10 – 12	1814	+
OF ENTRY	14 – 3	1860	
In 1863 .	–	1863	
OF EXIT	6 – 3	1879	
Duration before 1863		3	
Duration of Annuity		13	
Age at Entry .		45	
Age in 1863 .			
Age at Exit .			

MODE OF EXIT (D.)

REMARKS.

78

(Select Tables:—"Old" and "Combined").
(Aggregate Tables:—"Old" and "Combined").

(b)

CS OLD ANNUITIES.

NO. 1540 £ 75

M. LIFE { Marner, Silas,

DATE—	D. M.	YEAR.	
OF BIRTH	10 – 12	1814	
OF ENTRY	17 – 6	1862	
In 1863 .	–	1863	
OF EXIT	6 – 3	1879	
Duration before 1863		1	
Duration of Annuity		16	
Age at Entry .		48	
Age in 1863 .			
Age at Exit .			

MODE OF EXIT (D.)

REMARKS.

011

(Select Tables only:—"Old" and "Combined").

(c)

CS NEW ANNUITIES. A

NO. 1691 £ 25

M. LIFE { Marner, Silas.

DATE—	D. M.	YEAR.	
OF BIRTH	10 – 12	1814	+
OF ENTRY	18 – 1	1867	
OF EXIT	6 – 3	1879	
Duration of Annuity		12	
Age at Entry		52	
Age at Exit .			

MODE OF EXIT (D.)

REMARKS.

78

(Select Tables:—"New" and "Combined."—Aggregate Tables:—"New" only).

(d)

X NEW ANNUITIES. X

NO. 1699 £ 20

M. LIFE { Marner, Silas.

DATE—	D. M.	YEAR.	
OF BIRTH	10 – 12	1814	+
OF ENTRY	25 – 2	1867	
OF EXIT	6 – 3	1879	
Duration of Annuity		12	
Age at Entry .		52	
Age at Exit .			

MODE OF EXIT (D.)

REMARKS.

78

(Excluded from all Tables.)

NOTES AS TO THE

PRINCIPLES AND METHODS ADOPTED

FOR

CLASSIFYING AND TABULATING THE DATA.

ASSURANCE EXPERIENCE.

BY

THOMAS G. ACKLAND, F.I.A.,

Hon. Official Supervisor.

ASSURANCE EXPERIENCE.

NOTES AS TO THE PRINCIPLES AND METHODS ADOPTED FOR CLASSIFYING AND TABULATING THE DATA.

I. PRELIMINARY PROCESSES OF CLASSIFICATION.

(1). The Assurance cards received from the 60 English and Scottish contributing Offices, as entered up by them in accordance with the Instructions issued by the Joint Committee (*see* Appendix A), were first sorted according to sex, by separating the white cards (Male lives) from the pink cards (Female lives); and the cards in respect of each sex were then sorted according to the several Classes of Assurance, which were distinguished by initial letters entered on the line marked "Class," as specified in clause (12) of the Instructions. Specimens of the form of card employed (reduced in size) are given in Tables V and VI, pp. 56, 57.

II. AS TO THE METHODS ADOPTED FOR DETERMINING THE TABULAR AGE AT DATE OF ASSURANCE.

(2). Before determining the method to be adopted for arriving at the tabular age at date of assurance, investigations were made as to the interval subsisting, in the case of each assurance, between the date of assurance and the last preceding birthday. These investigations were made separately in respect of each of the eight groups shewn below; and in Appendix C, are given the relative numbers, tabulated in respect of each month of interval, and reduced to a total of 10,000 in each case. The average intervals were as follows:—

TABLE I.

Section of Experience.	Whole-Life Assurances.		Endowment Assurances.	
	Male Lives.	Female Lives.	Male Lives.	Female Lives.
	Months.	Months.	Months.	Months.
English	7·332	6·908	7·847	7·575
Scottish	7·687	7·326	7·985	7·855

(3). From the Table in Appendix C, the extent of the errors involved in treating the experience as if all assurances were effected at the nearest ages at date of assurance can be ascertained. Thus, in the Male Experience, English Section, Whole-Life Assurances, 3,647 lives out of 10,000 would be tabulated as assuring on the last birthdays, and the remaining 6,353 lives on the next birthdays. This would give an average interval between the date of assurance and the last preceding birthday of ·6353 of a year, or say 7·624 months. The true average interval being 7·332 months, the effect of the method would be to overstate the age at date of assurance by about 8¾ days. If, on the other hand, the age at date of assurance were assumed to be the age last birthday, or next birthday, with an assumed interval of six months, the deviation from the true age would be 1·332 months, or about 41 days. Thus the former method gives the closer approximation; and a similar result is shown by each of the Tables given. The age at date of assurance was therefore taken throughout as that attained upon the birthday nearest to the date of assurance, as determined by a comparison of the dates recorded upon the cards. The age thus deduced was that termed by Dr. Sprague (*J.I.A.* XXXI. 208) the "Commencing Age." It was found that the most practical way of arriving at this result was first to modify the year of birth, so that the difference between the calendar year of assurance and the modified year of birth would give in all cases the nearest age at date of assurance. The modification of the year of birth was given effect to by marking the recorded year + or − in certain cases, determined by the rules set out in Appendix D. Those cases in which the data as to birth were defective were set aside, and dealt with by the methods set out in Appendix E. The aggregate number of these cases did not exceed about 3 per cent. of the Male Experience, and about 6 per cent. of the Female Experience.

III. AS TO THE METHODS OF RECORDING AND TABULATING THE DURATIONS.

(A). AS AT ENTRY UNDER OBSERVATION.

(4). In regard to all "Old" assurances, the duration upon the anniversary in 1863 was recorded, being the duration of the policy at the time of entry under observation. This duration was necessarily integral and exact. As the "New" assurances came under observation from the original date of entry (duration 0), it was not necessary to record specifically in this class the duration as at entry under observation.

(B). AS AT EXIT.

(5). The durations as at exit marked upon the cards were as under:—

(*a*) **Deaths.**—The curtate duration, or the duration at the commencement of the policy year of death.

(*b*) **Existing.**—The exact integral duration upon the policy anniversary in 1893, when the life passed out of observation by expiry of the period of observation.

(*c*) **Withdrawals and Terminations.**—The curtate duration, or integral duration at the policy anniversary immediately preceding (or exactly according with) the cessation of the risk; together with a further record (see below), as locating the event within the policy year, so as to make provision for the fractional exposure within the year of exit.

The detailed work of determining the integral and fractional durations to be so recorded upon the cards was found to be most efficiently and expeditiously performed by methods of sorting in several stages, as specified in detail in Appendix F.

(6). **Fractional Exposure of Withdrawals.** — Having regard to the desirability that the tabulated records should furnish full information as to the cases of withdrawal, the Joint Committee determined to have prepared separate statistical records of the withdrawals, in such manner as to show, for each age at entry, or group of ages at entry, the actual incidence of the cases withdrawing in each year of assurance. This was carried into effect by sorting and tabulating the withdrawals in each policy-year into *four* groups of fractional duration, 0-2 Months; 2-6 Months; 6-8 Months; and 8-12 Months; the central points of which groups represent the durations (inclusive of the days of grace) of the lapses arising respectively in the first, second, third and fourth quarter of the year. The cases of withdrawal comprised in these four groups were conveniently designated (with reference to their average or central durations expressed in months), W(1), W(4), W(7), and W(10), respectively. An example of the form in which the withdrawals were thus scheduled is given in columns (1) to (6) of Table III (p. 45).

(7). The data thus scheduled were employed in determining the *tabular* duration of the withdrawals, that is to say, the assumed terms of their respective exposures. As it was desired to avoid fractions in these terms of exposure, the tabular durations to be deduced must necessarily be throughout integral; and the question

to be decided thus resolved itself into the determination of the principles upon which the withdrawals, falling in a given year, scheduled in the four groups of fractional duration above specified, should be referred to the beginning, or to the end, of the policy year, respectively, for the term of their tabular exposures.

(8). In the first place, it may be stated that some confusion was caused by the fact that varying interpretations had been placed upon the Instructions to the Offices, with the result that it appeared, upon examination of the cards contributed by the several contributing Companies, that 25 had included a uniform period as the days of grace; in one case, 15 days; and in the remainder, 30 days, or one calendar month; 29 had throughout excluded the days of grace, whilst in the case of the remaining 6 Offices no definite rule was, from an inspection of the cards, readily ascertainable, the cases including, probably, varying periods of non-forfeiture (dependent upon the value of the policy) during which the assurance was maintained in full force. As, however, the Joint Committee had decided (*see* Appendix B, clause 10) that the data should be so tabulated as to include the days of grace (almost universally one calendar month or thirty days) within the period of observation, it became necessary to find some special method of estimating, in the case of withdrawals, the fractional period of observation in the last policy year, as affected by the days of grace.

(9). For the purpose of ascertaining the most satisfactory method of dealing with this question, a body of cards was set aside and made the subject of a preliminary investigation. The cards selected were those relating to Male lives born in the year 1846 included in the class of "New" Whole-Life Assurances, and from these selected cards the Table given in Appendix G was constructed, showing in detail the distribution of the withdrawals in each of the first 30 years of assurance. This Table formed the basis of certain experimental methods and groupings, in order to select that method which appeared to be the most expeditious and accurate for the purpose. The cases recorded as passing out of observation at the *exact* points, 0, 3, 6 and 9 months after the policy anniversary, as well as those at the *exact* points, 1, 4, 7 and 10 months, were tabulated separately, as showing (with the exception of such surrenders as were effected at those precise points) the lapses in respect of which the days of grace had been excluded from, or included within, the recorded period of observation, as derived from the dates on the cards.

(10). The Nearest Duration Method being known as the most facile in operation, some modification of that method was sought

which should, without introduction of complexity, be appropriate and substantially accurate in the particular circumstances of this experience. The Nearest Duration Method, as is well known, lends itself admirably to the case where the days of grace are throughout excluded from the period of exposure; for the large body of withdrawals which then occur (by lapse) precisely at the beginning or end of a year are correctly recorded; those withdrawing after 6 months, being equally distributed, are also correctly recorded upon the average; and those withdrawing at 3 and 9 months, which may fairly be considered as approximately equal in number, are so treated as practically to introduce compensating errors.

(11). Where, however, as in the present case, it is desired to include the days of grace in the period of exposure, it is clear that the Nearest Duration Method is not equally applicable. This is illustrated by Table II (p. 44), where the integral duration, as tabulated by the Nearest Duration Method, is compared, in each successive policy year, with the exact duration as actually experienced the selected data employed being that contained in Appendix G, already referred to. It will be observed that, while the true duration of the withdrawals is, in the aggregate, represented with close accuracy by the Nearest Duration Method, the durations in individual policy years, and especially in the early years, are considerably distorted, the deviation amounting to 40 per cent. in excess in the first year of assurance, 12 per cent. in defect in the second year of assurance, 11 per cent. in defect in the third year of assurance, and 10 per cent. in defect in the fourth year of assurance. In considering the effect of these deviations from the true exposures upon the total number exposed to risk, and upon the resulting rates of mortality, consideration must of course be given to the total number of cases entering upon each policy year (*see* column (2) of Table II); and the inclusion of this large body of cases, as might be expected, materially reduces the proportionate error in the resulting numbers exposed to risk. Bearing in mind, however, the importance of accurate data in forming conclusions as to the effect of selection, and the true rate of withdrawal in the early years of assurance, it seemed most desirable that some method should be employed which should give a closer approximation to the true duration of the withdrawals, considered as a separate class, in the early years of assurance.

(12). The experimental investigation referred to above led to the conclusion that, for the first 30 years of assurance, the following modification of the Nearest Duration Method would

give results closely according with the actual exposures of the cases:—

MODIFIED NEAREST DURATION METHOD.

(i) Refer cases whose fractional durations fall in the period 0-6 months (marked W(1) and W(4) on the cards) to the beginning of the policy-year current at exit;

(ii) Refer cases whose fractional durations fall in the period 6-8 months, both inclusive (marked W(7) on the cards), alternately to the beginning and end of the policy-year current at exit;

(iii) Refer cases whose fractional durations fall in the period 8-12 months (marked W(10) on the cards), to the end of the policy-year current at exit; and

(iv) Transfer, from the beginning to the end of the policy-year, further cases, equal in number to one-twelfth of the total number of withdrawals falling in the year.

(13). The *rationale* of this method, and its special applicability to the circumstances of the present experience, as well as an investigation of the amount of error involved in the deduced durations, and an example of the practical application of the method, are fully set out in Appendix M.

(14). The withdrawals having, as stated in § (6), been throughout scheduled, for statistical purposes, in four groups, 0-2 Months, 2-6 Months, 6-8 Months, and 8-12 Months, according to their fractional durations at exit, the material was at once available for deducing the tabular durations, by the application of the Modified Nearest Duration Method. Referring to the details of the method, as set forth in section (12) above, it will be observed (i) that the cases referred to the beginning of the policy-year are each given a tabular duration (in that year) of 0 months; (ii) that the cases referred alternately to the beginning and end of the policy-year, are, one with another, given an average tabular duration of 6 months, or half a year; (iii) that the cases referred to the end of the policy-year are each given a tabular duration of 12 months, or one year; and, finally, (iv) that the transference of one-twelfth of the whole number of withdrawals falling in the year, from its commencement to its close, is equivalent to the addition of a further exposure of one month, in respect of each withdrawal. The tabular exposures of the whole of the withdrawals falling in the policy-year, will thus be equal to

$$0[W(1)+W(4)]+\tfrac{1}{2}W(7)+W(10)+\tfrac{1}{12}W,$$

which reduces to

$$\tfrac{1}{2}W(7)+W(10)+\tfrac{1}{12}W,$$

where W(7) and W(10) represent the withdrawals scheduled as falling in the fractional periods 6-8 months, and 8-12 months, respectively; and W is the total number of withdrawals in the year. The statistical records of the withdrawals in these groups thus formed a convenient basis for the calculation, by the above arithmetical processes, of the tabular exposures, without further reference to, or sortings of, the cards. The form of schedule employed for the statistical record of the withdrawals, according to age at entry, year of assurance, and fractional grouping of durations, and also for the calculation of the tabular exposures of such withdrawals, is given in Table III, from which it is hoped that the whole of the operations followed will be rendered perfectly clear. A note at foot of the Table sets forth the rules practically followed in ascertaining the tabular durations by the above formula, so as to avoid the introduction of fractions in the computed exposures.

(15). Comparing now the years of risk of the cases of withdrawal, as estimated by the Modified Nearest Duration Method, with the true fractional period of risk, it will be seen, by reference to Table II, that the actual deviations arising over the 4,688 withdrawals included in Appendix G amount *in the aggregate* to twelve years and nine months of risk, giving a deviation of about one day in each case. The effect of the method is, however, even better seen when applied to the individual years of assurance; and it will be observed, on reference to columns (3), (6) and (7) of Table II, that the durations, as tabulated in individual years of assurance by the method now proposed, are practically indistinguishable from the exact durations of the withdrawals. A comparison of columns (7) and (5) will also illustrate the superiority of the method now proposed over the Nearest Duration Method, in the illustrative experience here investigated.

(16). In confirmation of the general applicability of the method to different classes of the experience, and to assurances of different durations, further investigations were made, the results of which are given in Appendices H to L, and relate severally to

(1). Whole Life Participating "New" Assurances, Male Lives, born in 1862.	Appendices H and J.
(2). Whole Life Participating "New" Assurances, Male Lives, born in 1816.	
(3). Whole Life Participating "New" Assurances, Female Lives, born in the years 1850 to 1865.	

TABLE II.

WHOLE-LIFE ASSURANCES.
PARTICIPATING AND NON-PARTICIPATING.
MALE LIVES—BORN IN 1846.
NEW ASSURANCES—EFFECTED 1863-1892.

Comparative Statement of Fractional Duration of Withdrawals, as estimated by the Exact Duration Method, the Nearest Duration Method, and the Modified Nearest Duration Method, also of the Number of Cases (whether subsequently withdrawn or not) entering upon each Year of Assurance

Curtate Duration	Number of Cases Entering on Year of Assurance	Exact Duration Method*		Nearest Duration Method †			Modified Nearest Duration Method			Curtate Duration
		Duration		Duration	Deviation		Duration	Deviation		
		Years	Months	Years	Years	Months	Years	Years	Months	
0	15,266	211	7	296	+84	5	207	— 4	7	0
1	14,585	189	3	166	—23	3	189	— 0	3	1
2	13,224	162	7	144	—18	7	163	+ 0	5	2
3	12,216	116	9	105	—11	9	118	+ 1	3	3
4	11,409	97	3	93	— 6	3	99	+ 1	9	4
0—4	66,700	777	5	804	+26	7	776	— 1	5	0—4
5	10,726	80	9	74	— 6	9	82	+ 1	3	5
6	10,031	61	8	55	— 6	8	63	+ 1	4	6
7	9,401	64	10	68	+ 3	2	67	+ 2	2	7
8	8,780	60	3	56	— 4	3	61	+ 0	9	8
9	8,164	39	2	36	— 3	2	39	— 0	2	9
5—9	47,102	306	8	289	—17	8	312	+ 5	4	5—9
10	7,536	35	8	34	— 1	8	36	+ 0	4	10
11	6,984	31	8	30	— 1	8	32	+ 0	4	11
12	6,410	37	4	39	+ 1	8	40	+ 2	8	12
13	5,811	31	4	31	— 0	4	33	+ 1	8	13
14	5,223	20	4	19	— 1	4	20	— 0	4	14
10—14	31,964	156	4	153	— 3	4	161	+ 4	8	10—14
15	4,676	16	2	15	— 1	2	17	+ 0	10	15
16	4,116	10	10	11	+ 0	2	11	+ 0	2	16
17	3,517	10	2	11	+ 0	10	11	+ 0	10	17
18	2,933	9	4	9	— 0	4	11	+ 1	8	18
19	2,417	10	1	10	— 0	1	10	— 0	1	19
15—19	17,659	56	7	56	— 0	7	60	+ 3	5	15—19
20	1,915	11	0	11	...	...	12	+ 1	0	20
21	1,486	4	0	5	+ 1	0	4	...	...	21
22	1,107	4	5	5	+ 0	7	5	+ 0	7	22
23	813	0	5	0	— 0	5	0	— 0	5	23
24	559	1	5	1	— 0	5	1	— 0	5	24
20—24	5,880	21	3	22	+ 0	9	22	+ 0	9	20—24
TOTALS	169,305	1,318	3	1,324	+ 5	9	1,331	+12	9	TOTALS
(1)	(2)	(3)		(4)	(5)		(6)	(7)		(8)

* Cases of Withdrawal at the precise points 0, 8, 6, 9 (as recorded), being treated throughout as of durations 1, 4, 7, 10 respectively.

† Cases recorded as Withdrawals at the precise point 6 being treated as of duration 7, and classed with those falling in the second half of the year.

TABLE III.

WHOLE-LIFE ASSURANCE EXPERIENCE 1863-1893

SELECT TABLES **MALE LIVES**

Table of Distribution of WITHDRAWALS, *with Calculation of* FRACTIONAL EXPOSURES

CLASS **O** **Age at Assurance 26=[x]** SECTION **P**

"New" Assurances — With Profits

CURTATE DURATION	DISTRIBUTION OF WITHDRAWALS					FRACTIONS OF EXPOSURE		FRACTIONAL EXPOSURE	BALANCE
	TOTAL	0–2 months	2–6 months	6–8 months	8–12 months	$\frac{1}{2}$ W(7)	$\frac{1}{12}$ W	(6)+(7)+(8)	(2)−(9)
t	W	W(1)	W(4)	W(7)	W(10)	$\frac{1}{2}$(col 5)	$\frac{1}{12}$(col 2)	$W^{(b)}_{[x]+t}$	$W^{(a)}_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0	617	6	124	364	123	182	51	356	261
1	1,711	1,366	67	179	99	90	142	331	1,380
2	1,015	743	71	108	93	54	84	231	784
3	697	445	85	81	86	41	58	185	512
4	503	290	70	54	89	27	42	158	345
5	399	238	49	50	62	25	33	120	279
6	277	143	52	36	46	18	23	87	190
7	278	147	39	32	60	16	23	99	179
8	211	118	25	27	41	14	17	72	139
9	178	97	29	25	27	13	15	55	123
0—9									
10	159	74	31	19	35	10	13	58	101
11	144	80	28	14	22	7	12	41	103
12	123	57	30	15	21	8	10	39	84
13	119	63	21	18	17	9	10	36	83
14	78	42	12	11	13	6	6	25	53
15	80	35	16	14	15	7	6	28	52
16	66	34	12	5	15	3	5	23	43
17	67	35	8	9	15	5	5	25	42
18	57	25	10	10	12	5	5	22	35
19	37	21	4	1	11	1	3	15	22
10—19									
20	50	25	10	5	10	3	4	17	33
21	26	12	5	3	6	2	2	10	16
22	21	10	5	...	6	...	2	8	13
23	24	12	1	1	10	1	2	13	11
24	12	2	5	3	2	2	1	5	7
25	7	3	1	1	2	1	...	3	4
26	2	1	...	1	...	1	...	1	1
27	8	2	1	2	3	1	...	4	4
28	5	1	1	2	1	1	...	2	3
29	2	...	...	...	2	...	...	2	...
20—29									
0—29	6,973	4,127	812	1,090	944	553	574	2,071	4,902

NOTE.—The integral numbers included in column (7) were deduced by dividing the corresponding numbers in column (5) by 2, *all odd numbers being first increased by unity.* The integral numbers in column (8) were deduced by dividing the corresponding numbers in column (2) by 12, *all remainders up to 8 inclusive being neglected,* and the integral quotient increased by unity, where the remainder exceeded 8. The effect of these two processes (taken together) was very slightly to increase the numbers in column (9), and to reduce those in column (10), the average increase being about 1 *case in* 24 *durations.* The total of column (9), if correctly deduced, is $=(944+545+581\frac{1}{12})=2,070\frac{1}{12}$.

(4). Endowment Assurances, "New," Participating and Non-Participating, Male Lives, born in 1846 and 1862. Appendices K and L.

An examination of these data and results will show that the withdrawals were similarly distributed in each of these classes and periods of life, and that the method adopted for tabulating the exposures gave in each case closely approximate results.

(17). **Fractional Exposure of Terminations.**—The second class of cases requiring treatment in regard to fractional periods of exposure in the year of exit were the "Terminations." These were of five kinds:—(i) Maturities of Endowment Assurances; (ii) Expiration of the period of assurance in Temporary Assurances; (iii) Cessation of the risk on the surviving life by the death of the other life, in the case of Joint Life Assurances; (iv) Cessation of the risk under Contingent Survivorship Assurance policies, by the death of the counter-life; (v) Miscellaneous cases, under all classes of assurance, where the contract of assurance had been broken, and which had been reported as mode of exit "T." There were also other cases falling under classes (i) to (iv) through the class of assurance having been changed, subsequently to the issue of the policy, from Whole Life to one or other of the classes named above. (*See* clause 1 of supplementary Instructions to Offices, Appendix B.)

(18). (i) ENDOWMENT ASSURANCES.—In this class there were about 6,200 cases of Termination in the Select Tables, of which (*a*) 4,050 fell on a quinquennial birthday; (*b*) 1,350 fell on a policy anniversary; (*c*) 800 cases fell at miscellaneous dates, sometimes at other birthdays or at odd dates, showing them to be largely maturities paid in advance, but treated as maturities in the records on the cards. The cases in class (*b*) could at once readily have their exact integral duration recorded thereon. The cases of type (*c*) were sufficiently well dealt with by the ordinary application of the Nearest Duration Method. An exceptional difficulty, however, arose in the main class of cases of type (*a*), maturing upon the quinquennial birthday, consequent upon the application of the Nearest Age Method. For, considering the Terminations on the sixtieth birthday, these, since the tabular age at date of assurance is the "Nearest" age on such date, would fall either in the second half of the tabular year of age 59–60, or in the first half of the tabular year of age 60–61, according as the actual sixtieth birthday was before, or after, the then nearest policy anniversary. At each quinquennial age, therefore, the Terminations, falling as they would in adjacent halves of *different*

policy years, would not yield those measures of compensation within their own policy years which form the basis for the application of the Nearest Duration Method.

(19). An illustration may serve to render this somewhat important point clear. Let two endowment assurances be effected, one (*a*) at actual age $39\frac{3}{4}$, the other (*b*) at actual age $40\frac{1}{4}$; and let both assurances mature on the attainment of the 60th birthday. In both cases, the tabular, or nearest, age at entry would be 40; and the full durations of the assurances up to maturity would be $20\frac{1}{4}$ years in case (*a*), and $19\frac{3}{4}$ years in case (*b*). The maturities would thus fall in *different policy-years*, (*a*) in the first half of the 21st year, (*b*) in the second half of the 20th year. By the application of the Nearest Duration Method, both cases would be tabulated as passing out of observation after an integral duration of 20 years, at tabular age 60. In all cases of type (*a*) there would thus be a deficiency (to an extent in each case not exceeding six months) in the tabulated durations; and, in all cases of type (*b*), there would be a corresponding excess in the tabular durations; and these would tend to be compensatory in amount. As, however, the cases of excess exposure would arise in the policy-year *next preceding* that in which the cases of deficient exposure would arise, there would be no compensating corrections *in the same policy-year;* and, as the maturities arise (practically) at every fifth age only, the effect is that, at these particular ages, the exposures in the year immediately preceding are throughout overstated, whilst those in the following year are throughout correspondingly understated.

(20). From results of some tabulation of cases (*see* Appendix N), which matured upon the sixtieth birthday, it was ascertained that those falling in tabular year of age 59–60 had a mean duration of about 9 months; and that those falling in tabular year of age 60–61 had a mean duration of about 2 months; whereas, by the Nearest Duration Method, they would be treated as being under observation for 12 months and 0 months respectively in their policy-year of exit. To overcome this difficulty, in the former group one case out of every four maturities within the year was *deducted* from the exposed to risk for that year, as arrived at by the Nearest Duration Method; and in the latter group one case out of every six maturities within the year was *added* to the exposed to risk, as arrived at by the Nearest Duration Method.

(21). (ii) TEMPORARY ASSURANCES.—The few cases for terms other than complete years in case of temporary assurances were

dealt with by the ordinary application of the Nearest Duration Method.

(22). (iii) and (iv). JOINT LIFE AND CONTINGENT SURVIVORSHIP ASSURANCES.—These cases were also dealt with by the Nearest Duration Method.

(23). In the actual manipulation of all these cases, the system of stamping the duration upon the cards was carried out as follows :— The *curtate* duration was in each case stamped upon the card, and the card was thereafter marked (A) or (B), according as the case was to be deducted from the exposed to risk at the commencement, or close, of the current year of assurance.

(24). (v). MISCELLANEOUS CASES.—Where cases reported as mode of exit "T" arose specially in the Whole-Life Assurance Class, they were treated as Withdrawals, and sorted into the four groups there employed (*see* § (6) above).

(25). **Defective data as to Exit.**—In the few cases where the data as to Exit (whether by Death, Withdrawal, or Termination) were defective, it was found necessary, in order to determine the period of exposure for purposes of tabulation, to make certain assumptions, which, with the methods followed in carrying them into effect, are fully set out in Appendix O. The aggregate number of cases thus dealt with did not exceed about one-half per cent. of the total number of cases under observation.

IV. AS TO THE PRINCIPLES AND METHODS ADOPTED IN THE TREATMENT OF DUPLICATES.

(26). The general methods of dealing with the cases where more than one policy subsisted, in any class of assurance, upon the same life, during the period of observation, can best be described under the two separate headings—**Collocation of Duplicates**—which relates to the steps taken in order to bring together the several cards relating to the same person ;—and **Elimination of Duplicates**—which relates to the methods adopted in discriminating the Table or Tables to which the respective cards were required to contribute their data.

(27). **Collocation of Duplicates.** — It was found, by a preliminary investigation, that a general alphabetical arrangement of the names of the lives recorded on the cards would not be the most suitable and efficient for the bringing together of cards relating to the

same life. This arose partly from the large volume of the data, the Whole-Life Male Assurance Experience being recorded on upwards of 800,000 cards, contained in about 400 boxes. Slight variations of spelling in the names would, in so large a volume of data, frequently escape attention; "Lydgate," for instance, being in a different box from "Lidgate," and separated from it by a large number of cards. It was also found that, apart from varieties of spelling, cases of compound or alternative surnames (especially among the Peerage) were frequently met with; and these again could not be detected in a purely alphabetical arrangement. The method adopted proceeded by an arrangement of the cards according to date of birth, in strictly chronological order; and, after bringing together all duplicates thus detected, and setting aside doubtful cases for further examination, the cards *in each year of birth* were then sorted according to alphabetical order of name, and again examined for duplicates and doubtful duplicates. Full details of the methods followed in each of these processes of examination, and in dealing with cases of defective data as to birth, as well as the additional steps taken to detect duplicates in the case of Female lives, are set out in Appendix P.

(28). **Elimination of Duplicates.**—Having, by means of the various processes detailed under the heading "Collocation of Duplicates," brought together all the cards under each separate class of assurance relating to the same life, it still remained to select the card or cards the experience recorded on which was required (A) for Select Tables, (B) for Aggregate Tables. In view of the many complexities which must evidently have arisen in carrying out this process of selection in such a way as to obtain independently (should they be required)—

(*a*) Tables setting out the separate experience of

(i) Participating assurances effected before 1863, but not coming under observation until their anniversary in that year (Old P);

(ii) Non-participating assurances effected before 1863 (Old N);

(iii) Participating assurances effected since 1862, and coming under observation from their inception (New P);

(iv) Non-Participating assurances effected since 1862 (New N);

as well as—

(*b*) Tables setting forth the experience of two or more of these sections combined;—

the Joint Committee decided that duplicates should be eliminated within each of the four several sections upon the following principles:—

(A) SELECT TABLES.—One card only was selected to represent the experience of each life in respect of each "Age at date of assurance."

(B) AGGREGATE TABLES.—One or more cards were selected and dealt with in such a manner that each life was represented by only one card in respect of the period of risk recorded thereon, without regard to the "Age at date of assurance."

It was, however, decided that, in forming any combination of the four sections, such as "Old" assurances—Participating and Non-participating combined; Participating assurances—"Old" and "New"; Participating and Non-participating assurances—"Old" and "New"; the data for the component sections should be combined, without regard to the fact that certain of the lives might be then represented by concurrent periods of risk (with the duplicate records of death appertaining) derived from the several component sections in which such lives were included.

(29). The process of denoting how each card on the same life was to be employed in compiling the data, was that of distinctly marking each of the cards as follows :—

"S A" where the risk recorded on the card was required for the construction of *both Select and Aggregate Tables*;

"S" where the risk recorded on the card was required for *Select Tables only*, the period of risk being wholly duplicated by that already covered by other cards marked for use in the Aggregate Table;

"X" where the card was *not required for either Select or Aggregate Tables*, there being already another card at the same age at entry.

The cards representing unduplicated risks were unmarked; and these contributed their experience uniformly to both Select and Aggregate Tables.

(30). SPECIAL TREATMENT OF PARTIALLY DUPLICATED RISKS. In dealing with risks which overlapped, a plan hitherto adopted, in the construction of Aggregate Tables, has been to bring together the cards in an envelope, and record thereon the earliest date of entry, and the latest date and mode of exit. In lieu of this, the following plan was adopted. Each of the cards forming the risk was retained.

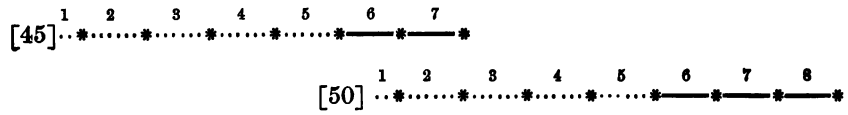
and each of the later cards (*i.e.*, all but the first) was marked with a "commencing duration," representing its nearest integral duration at the date when it came under independent observation, that is, at the date of exit of the immediately preceding case which was withdrawn during its currency.

(31). These cards were further stamped—the first case, "P D," and the remainder,—being those which bore the additional record of "commencing duration,—"*P.D.*" The cards marked "P D" were then employed in exactly the same manner as those marked "S A," and the unmarked cards representing unduplicated risks, these all contributing their experience uniformly to both Select and Aggregate Tables, either from the policy anniversary in 1863 (if "Old" assurances) or from the date of assurance (if "New" assurances). The cards marked "*P.D.*," however, while contributing their experience to Select Tables in precisely the same manner as above, contributed their experience to Aggregate Tables from the date of the "commencing duration" recorded thereon, that is from the date at which they came under independent observation; so that, in compiling the Aggregate Tables for the purposes of computing the numbers passing out of observation at each age and duration, each such card was employed in a similar manner to that of the cards marked "S A"; but, in computing the number of entrants under observation at each age and duration, the group in which such card was to be included was determined from the "commencing duration" marked thereon.

(32). This method of dealing with the cards had several specific advantages. (i) The labour involved in stamping the "commencing duration" was much less than that which would have been required to write up a fresh set of cards or envelopes embodying the complete period of risk,—especially as the original cards would still have been required in the formation of the Select Tables, whilst the fresh set of records would have had to be substituted in the Aggregate Tables. (ii) The method of compilation being by policy years, the particular incidence of the withdrawals (to meet which an appropriate method of treatment had been adopted) would have been disturbed, if the policy year current at exit had been reckoned, not from the date of the assurance last effected, but from the commencement of the period of continuous risk. (iii) As it was desired, later on, to construct a Table, or a series of Tables, from which the data in respect of certain of the earlier years of assurance should be excluded, and as the number of years to be so excluded could not at the outset be determined, it was considered preferable to adopt a system by which every portion

of the whole period of risk recorded was absolutely and rigidly included within the year of assurance of the policy under which it was observed.

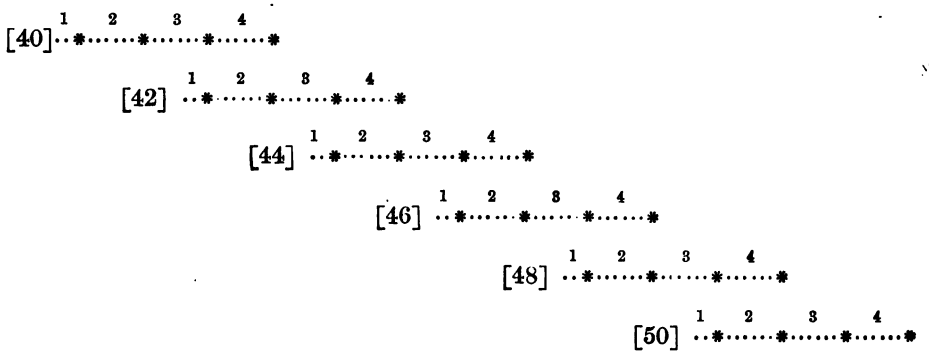
(33). In regard to this last point, the following explanation in detail may be of interest. In the combined-risk card, the true duration of each component risk (reckoned from entry) becomes merged, as it were, in one continuous period, and cannot be afterwards separately identified. Thus, to take a simple case, let two policies be effected by the same life, one at age 45, the other at age 50, the former policy being allowed to lapse after seven years' duration (or at age 52), while the latter policy is under observation until age 58. This may be set out graphically as follows :—



Here the period during which selection is considered to be in operation (assumed in this example to be five years) is indicated by dotted lines, and the subsequent "non-select" period by continuous lines. The figures in square brackets show the ages at selection, and the small figures, the years of assurance passed through in respect of each policy. If a combined-risk card were prepared in this case, the risk would be treated as a continuous one running from age 45 to age 58, and the case would be tabulated in all respects as if the risk were upon a single policy effected at age 45, and under observation for 13 consecutive years of duration ; and, if the first five years of risk were then excluded from this combined-risk card, the period remaining, from age 50 to 58, would be tabulated as the experience to be included in the non-select Table. But this latter period includes three years (from age 52 to 55) during which the life was, under the second policy (then alone in force), still within the period of selection. These years of risk do not appear to be properly included in the non-select Table, for if each constituent policy were separately "truncated," the whole of the experience marked in dotted lines would be excluded, and two portions of risk only would remain, one, under the first policy, running from 50 to 52, the other, under the second policy, running from 55 to 58.

(34). The above is typical of cases which very frequently arose in the experience. Another case might be taken, of a less common, but by no means impossible, type, to illustrate the point now under consideration. Let successive policies be taken out by the same life, at ages 40, 42 44, 46, 48 and 50; and let each of these policies be

allowed to lapse after four years' duration. This may be graphically shown as under:—



A combined-risk card in the usual form would set this out as a continuous risk from age 40 to 54, similar to that of a single policy passing through 14 consecutive years from entry; and, if the first five years' experience were then excluded, the assumed non-select risk would extend over the nine years from age 45 to 54. But the life *never at any time* passed into the non-select period, none of the policies having exceeded four years' duration; and it seems certain that no portion of the observed risk under these six policies should be included in the non-select Table.

(35). Thus, if successive partially-duplicated risks upon the same life are treated as continuous risks (by the employment of combined-risk cards or otherwise), and the select period be excluded from such continuous risks, as if in respect of single policies, it would appear that, in a large number of cases, the experience of the select period which it is desired to exclude will not in fact be excluded; and this must surely tend to affect the accuracy of the resulting truncated Table. In fact, in such case, the *first* selection is alone dealt with, any subsequent fresh selections, during the currency of the continuous risk, being altogether ignored. But the principle of first selection cannot even be said to be consistently followed out; for if, subsequently to the termination of the continuous period of risk, there were (after an interval) an independent period of risk upon the same life,—as, for instance, if in either of the examples previously cited, the life were also under observation from age 60 to 75,—the first five years of this independent period (being set out on a distinct card) would be excluded from the non-select experience; so that a later selection would in that case be taken into account. The difficulty of providing for the exclusion of the data in respect of the early period of selection might perhaps have been met by some

modification of the combined-risk card; but this does not seem to be practicable where, as in the present case, the term of years so to be excluded—whether extending over 5, 7, or 10 years, or some other term—could not be definitely determined at the outset.

(36). By the method of "commencing durations" actually adopted, in lieu of a combined-risk card or envelope, setting forth (in the first of the examples above cited) a continuous period of risk from age 45 to age 58, the constituent cards would both be employed, the first setting forth the period of risk from age 45 to age 52, while the second would be specially marked to indicate that the risk, originally entered upon at age 50, did not come under independent observation until the end of its second year of duration (or at age 52), and was then continued till age 58. This was effected by marking the second card distinctively with a "commencing duration" of 2 years. In the second example, in lieu of a combined-risk card setting forth a continuous period of risk from age 40 to age 54, the six constituent cards would all be employed, setting forth the successive periods of risk as under:—

TABLE IV.

No.	Age at Entry.	Commencing		Terminating	
		Duration.	Age.	Duration.	Age.
(1)	40	0	(40)	4	(44)
(2)	42	2	(44)	4	(46)
(3)	44	2	(46)	4	(48)
(4)	46	2	(48)	4	(50)
(5)	48	2	(50)	4	(52)
(6)	50	2	(52)	4	(54)

the special points of "commencing duration" being distinctively marked upon the cards (2) to (5) inclusive. The ages printed above in round brackets would not be recorded upon the cards, but are included in the above example to show how the continuous period of risk from age 40 to age 54 is "built up," as it were, by the constituent cards, each representing a partial risk. The "terminating durations" are simply the durations at exit of the individual policies upon the life, recorded in ordinary course upon the cards. It will be observed that this plan proceeds upon the principle of building up the continuous term of risk by the inclusion of each of its several component parts, with the great advantage that what may be termed the "articulation"

of the cases is directly under observation, as each constituent policy enters into the experience at a recorded point in its own history, instead of being artificially merged (as in the combined-risk card or envelope) with other policies of different durations. By this means, it is quite easy to provide from the outset with equal accuracy, both for the exclusion of the first t years of experience following the date of selection (whatever value may ultimately be given to t) in respect of every individual policy entering into the experience, and also for the elimination of duplicates in the truncated or non-select Table, arising from such modification of the experience.

(37). EXAMPLE ILLUSTRATING WHOLLY DUPLICATED RISKS. In Table V is given a typical example of the general principles of the distinctive markings employed in wholly duplicated cases, in respect of four policies effected on the same life and in the same section (Whole-Life Participating "Old" Assurances). The life in question, born on 7th August, 1826, took out four several policies at nearest ages 23, 29, 29, and 34, the first three of which were current till death on 29th February, 1888, whilst the fourth was withdrawn on 31st December, 1864, after an exposure (within the period of observation) of 1 year and 3 months only. Of the four cards relating to this life, the first (*a*) would be marked "S A," since the period of risk it represents covers the whole period for which the life was exposed in both Select and Aggregate Tables. The second card (*b*) and the third (*c*) would be marked "S," since they represent periods of risk which enter into the Select Tables at different entry ages, but are already represented in the Aggegate Tables by card (*a*). Finally, the card (*d*), which represents a period of risk duplicated by the periods of risk of the other cards in both Select and Aggregate Tables, would be marked "X," for exclusion from both Tables.

(38). EXAMPLE ILLUSTRATING PARTIALLY DUPLICATED RISKS. In the case of partially duplicated risks, a similar practical illustration will perhaps make the actual method more readily intelligible. In Table VI are set out particulars of four policies effected in the Whole-Life Participating "New" Section upon the same life. The first (*a*), effected at nearest age 20, withdrew after seven years' duration; the second (*b*), effected at nearest age 22, withdrew after four years; the third (*c*), effected at nearest age 25, withdrew after eight years; and the fourth (*d*), effected at nearest age 31, passed out of observation by later death. The card (*b*) would in the first place be marked "S" for inclusion in the Select experience, but as the period of risk under this case is entirely covered by that represented

TABLE V.

DISTINCTIVE MARKING OF DUPLICATES.

"WHOLLY DUPLICATED RISKS."

S. H. *Old Policies.* (a)

NO. 2356 £ 100

CLASS 0 PROFIT OR NOT P.

LIFE { *Latimer,* / *Darsie.*

DATE—	D.	M.	YEAR.	
OF BIRTH	7	8	1826	+
OF ENTRY	1	1	1850	
In 1863	–		1863	
OF EXIT	29	2	1888	
Duration before 1863			13	
Duration of Policy			38	
Age at Entry			23	
Age in 1863				
Age at Exit				

MODE OF EXIT (D.)

REMARKS.

L

(Select and Aggregate Tables.)

S. *Old Policies.* (b)

NO. 5690 £ 150

CLASS 0 PROFIT OR NOT P.

LIFE { *Latimer,* / *Darsie.*

DATE—	D.	M.	YEAR.	
OF BIRTH	7	8	1826	
OF ENTRY	25	3	1855	
In 1863	–		1863	
OF EXIT	29	2	1888	
Duration before 1863			8	
Duration of Policy			32	
Age at Entry			29	
Age in 1863				
Age at Exit				

MODE OF EXIT (D.)

REMARKS.

L

(Select Tables only.)

X *Old Policies.* (c)

NO. 6553 £ 250

CLASS 0 PROFIT OR NOT P.

LIFE { *Latimer,* / *Darsie.*

DATE—	D.	M.	YEAR.	
OF BIRTH	–			
OF ENTRY	6	8	1855	
In 1863	–		1863	
OF EXIT	29	2	1888	
Duration before 1863				
Duration of Policy				
Age at Entry			29	
Age in 1863				
Age at Exit				

MODE OF EXIT (D.)

REMARKS.

Age at Entry next Birthday, 29.

A

(Excluded from all Tables.)

S. *Old Policies.* (d)

NO. 12542 £ 500

CLASS 0 PROFIT OR NOT P.

LIFE { *Latimer,* / *Darsie.*

DATE—	D.	M.	YEAR.	
OF BIRTH	7	8	1826	
OF ENTRY	29	9	1860	
In 1863	–		1863	
OF EXIT	31	12	1864	
Duration before 1863			3	
Duration of Policy	(4)		4	
Age at Entry			34	
Age in 1863				
Age at Exit				

MODE OF EXIT (W.)

REMARKS.

b

(Select Tables only.)

NOTE.—The duration (4) marked on card (*d*) indicates that the case is to be included in the group of Withdrawals W (4), the fractional exposure in the policy-year of exit lying between 2 and 6 months.

TABLE VI.

DISTINCTIVE MARKING OF DUPLICATES.

"PARTIALLY DUPLICATED RISKS."

P. D. *New Policies.* (a)

NO. 4160 £ 150

CLASS 0 PROFIT OR NOT P.

LIFE { *Goldsmith,* / *Oliver, W.*

DATE—	D.	M.	YEAR.	
OF BIRTH	4	6	1850	
OF ENTRY	5	6	1870	
OF EXIT	9	7	1877	
Duration of Policy (1)			7	
Age at Entry .			20	
Age at Exit .				

MODE OF EXIT (W.)

REMARKS.

128

(Select and Aggregate Tables—from Entry.)

S. *New Policies.* (b)

NO. 6537 £ 250

CLASS 0 PROFIT OR NOT P.

LIFE { *Goldsmith,* / *Oliver, W.*

DATE—	D.	M.	YEAR.	
OF BIRTH	4	6	1850	
OF ENTRY	31	1	1872	
OF EXIT	13	10	1876	
Duration of Policy (10)			4	
Age at Entry .			22	
Age at Exit .				

MODE OF EXIT (W.)

REMARKS.

128

(Select Tables only—from Entry.)

P. D. *New Policies.* (c)

NO. 10491 £ 100

CLASS 0 PROFIT OR NOT P.

LIFE { *Goldsmith,* / *Oliver, W.*

DATE—	D.	M.	YEAR.	
OF BIRTH	4	6	1850	
OF ENTRY	1	3	1875	
OF EXIT	15	9	1883	
Duration of Policy (7)			8	A 2
Age at Entry .			25	
Age at Exit .				

MODE OF EXIT (W.)

REMARKS.

128

(Select Tables from Entry.— Aggregate from Duration 2.)

P. D. *New Policies.* (d)

NO. 21356 £ 500

CLASS 0 PROFIT OR NOT P.

LIFE { *Goldsmith,* / *Oliver, W.*

DATE—	D.	M.	YEAR.	
OF BIRTH	4	6	1850	—
OF ENTRY	10	12	1880	
OF EXIT	7	3	1892	
Duration of Policy			11	A 3
Age at Entry .			31	
Age at Exit .				

MODE OF EXIT (D.)

REMARKS.

128

(Select Tables from Entry.— Aggregate from Duration 3.)

NOTE.—The durations (1), (7), and (10), marked on cards (*a*), (*c*), and (*b*) respectively, indicate that the cases are to be included in the groups of Withdrawals, W (1), W (7), and W (10), the fractional exposures in the policy-year of exit falling within the periods 0—2 months, 6—8 months, and 8—12 months respectively.

by the card (*a*), the card (*b*) would for the moment be set aside, and the series of partially duplicated risks upon the life would be made up of the cards (*a*), (*c*) and (*d*). The card (*a*), representing the earliest entrant, would be marked "P D," and each of the cards (*c*) and (*d*) "*PD*," while the two latter cards would also respectively be marked "A...2..." and "A...3...," to indicate the points of nearest duration from entry at which they came under independent observation, as distinct risks, in the data for Aggregate Tables. Each of the cards (*a*), (*b*), (*c*) and (*d*), would enter into the Select Tables for the full term during which they were respectively under observation; but, for purposes of the Aggregate Tables, the first case (*a*) would be represented for the whole currency of its risk from age 20 to 27; the case (*b*) would be excluded entirely from the Aggregate Tables; the case (*c*) would contribute its experience from the expiration of its second year of duration (or from age 27) up to its withdrawal at age 34; and the case (*d*) would contribute its experience from the expiration of its third year of duration (or from age 34) until terminated by death. The continuous term or risk from age 20 to 27, under case (*a*); from age 27 to 34, under case (*c*); and from age 34 to the end of life, under case (*d*); would thus be correctly represented by the combination of these three constituent cards.

(39). The detailed Rules employed in the practical work of distinctively marking the cards representing duplicates are set out in Appendix Q. These will be found precisely to give effect to the principles enunciated above for the elimination of duplicates, and for the recording of partially duplicated risks.

V. AS TO THE METHODS ADOPTED IN THE TABULATION OF THE DATA.

(40). After the process of the elimination of duplicates, the cards representing Whole-Life Assurances (Male Lives) were sorted into the following three groups:—

"S and A" Group. Cards required for both *Select* and *Aggregate* Tables, comprising: (i) Unmarked cards, representing unduplicated risks; (ii) Cards marked "S A"; (iii) Cards marked "P D"—all of which came under observation without any modification as to the commencement of the period of risk.

"*PD*" Group. Cards so marked, which entered into the data for *Select* Tables from the commencement of the period of observation, but, for *Aggregate* Tables, from the "commencing duration" specially recorded upon the card.

"S" Group. Cards so marked, and required for *Select* Tables only.

(41). The cards in each of these three groups were then divided and further sub-divided

(i) according to Section :—

New Assurances { "New P," With Profits ;
"New N," Without Profits ;

Old Assurances { "Old P," With Profits ;
"Old N," Without Profits.

(ii) according to Mode of Exit :—

(θ) Death ;

(ϵ) Existing ;

(w) Withdrawal ; this latter group being again sub-divided according to the fractional duration recorded on the card :—

W(1) Duration 0—2 months (average, 1 month) ;
W(4) „ 2—6 „ („ 4 months) ;
W(7) „ 6—8 „ („ 7 „) ;
W(10) „ 8—12 „ („ 10 „) ;

(iii) according to Age at Entry ;

(iv) according to the integral Duration at Exit as marked on the cards, that is, for Existing cases the exact duration, for Deaths and Withdrawals the curtate duration.

The cards representing Endowment Assurances, Male Lives, and those for Female Lives in the two main classes of Assurance, were similarly classified.

(42). ENUMERATING CARDS. EMERGENTS.—The numbers in each of the packets finally arrived at were then counted, and recorded as *Emergents* on "Enumerating cards" of the form given in Table VII. Each of these Enumerating cards presented, in a concise form, a record of the number of cases passing out of observation in ten, or eleven, successive years of duration, in respect of the particular class of assurance, group for tabulation, age at entry, and mode of exit, specified on the heading of the card. By the employment of these cards the original data for Whole-Life Assurances, recorded on 735,079 cards (not counting those excluded for various reasons), were reduced to 12,950 Enumerating cards for subsequent use ; whilst the original 140,414 cards, in the class of Endowment Assurances, were similarly collected on 4,500 Enumerating cards. To prevent the possibility of any of these cards being lost or overlooked, they were numbered consecutively at foot by an automatic numerator, and a record kept.

TABLE VII.

ENUMERATING CARDS.

EMERGENTS.

MALES.

CLASS 0. New P. S. & A.

Age at Entry 38

Mode of Exit Death

Duration	*Number.*		*Initials.*	*Initials.*
10.				
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
19.				
0.				
		11203		

ENTRANTS.

MALES.

CLASS 0. Old P. S.

Age at Entry 47

Mode of Exit W(1)

Duration	*Number.*		*Initials.*	*Initials.*
20.				
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
29.				
0.				
		1310		

Similar cards were employed, in the Whole-Life and Endowment Assurance Classes, for each of the following Sections:—Old P, New P, Old N, New N; for the following groups (for tabulation of Select and Aggregate Tables) in each Section:—S and A, S, *PD*; and for the following cases of Emergents in each group:—

	Withdrawal.		Terminations.
Death.	W(1), W(4), W(7), W(10).	Existing.	$T^{(a)}$, $T^{(b)}$.

The cards marked $T^{(a)}$ gave particulars of the cases of Termination whose tabular exposures ended at the *commencement* of the year of assurance current at exit. Those marked $T^{(b)}$ recorded the particulars of the cases whose tabular exposures extended to the *end* of the year of assurance current at exit.

The ENTRANTS, in respect of Old Assurances, came under observation at the recorded durations in 1863; and in respect of New Assurances, from original entry (Duration 0); excepting only for Old and New cases in the group *PD*, when employed in the construction of Aggregate Tables, which came under observation from the "commencing duration" as marked upon the original data card after the letter A....

TABLE VIII.

ABSTRACT OF DATA CARDS.—EMERGENTS.

(*a*)

ASSURANCE DATA 1863-1893		M			
Assuring Age		CLASS O			
Duration		Withdrawals. N			
Age attained		(1)	(4)	(7)	(10)
S. & H.	OLD				
	NEW				
	O. & N.				
P.D.	OLD				
	NEW				
	O. & N.				
Aggregate Tables	*Old*				
	New				
	O. & N.				
S.	OLD				
	NEW				
	O. & N.				
Select Tables	*Old*				
	New				
	O. & N.				
Durations 0-29.					

(*b*)

ASSURANCE DATA 1863-1893		M		
Assuring Age		CLASS O		
Duration		Withdrawals ($w^{(a)}$).		
Age attained		P	N	P+N
S. & H.	OLD			
	NEW			
	O. & N.			
P.D.	OLD			
	NEW			
	O. & N.			
Aggregate Tables	*Old*			
	New			
	O. & N.			
S.	OLD			
	NEW			
	O. & N.			
Select Tables	*Old*			
	New			
	O. & N.			

Type (*a*).—These cards were employed for the first record, from the Enumerating cards, of the cases of Withdrawal, in the class of Whole-Life Non-Participating Assurances, Male Lives, O N (M); and from these cards the Schedules of Withdrawals were entered up. Similar cards were employed in the class O P, both for Males and Females; also in classes E P, E N.

Type (*b*).—Upon cards of this form were recorded the particulars of the cases of Withdrawal whose tabular exposures terminated at the *commencement* of the year of assurance current at exit, $W^{(a)}$, as deduced from the Schedules of Withdrawals. Similar cards, marked $W^{(b)}$, were employed for the record of the cases whose tabular exposures extended to the *end* of the year of assurance current at exit.

Cards similar to type (*b*) above, but with headings suitably modified, were employed for the record of the cases of Death (θ) and Existing (ϵ); also, in the class of Endowment Assurances, for the Terminations referred to the beginning, $T^{(a)}$, and to the end, $T^{(b)}$, of the policy year current at exit.

The cards containing the record of cases whose exposures terminated at the *commencement* of the policy year, were printed in a different colour from those containing the record of cases whose exposures terminated at the *end* of the policy year.

(43). ENTRANTS.—After the first counting, as Emergents, had thus been completed and verified, it was necessary to recount the cards as *Entrants*, for the purpose of recording the numbers which came under observation at each particular duration. They were therefore arranged according to duration at Entry under observation, in place of duration at Exit. The duration at entry under observation is as follows in the various groups :—

Form of Tabulation.	Group for Tabulation.	"Old" Assurances.	"New" Assurances.
Select Tables.	S and A.	The Duration in 1863, the commencement of the period of observation, as recorded upon the cards.	Duration 0, the cases being under observation from the original date of entry.
	S.		
	PD.		
Aggregate Tables.	S and A.		
	PD.	The "commencing Duration," specially marked on the card after the letter A...	

It will thus be seen that the cards included in the "*PD*" group had to be counted and recorded twice, once as Entrants for the purpose of Select Tables, from Duration 0 (New), or Duration in 1863, (Old); and again as Entrants for the purpose of Aggregate Tables, from the specially recorded "commencing Duration." The numbers Entered were then recorded on a further set of Enumerating cards specially marked "Entrants," and similarly classified to those employed for the record of the Emergents. A specimen of the Enumerating cards thus employed is given in Table VII. As a supplementary check upon the whole process of counting, the number of cases in each sub-division entering under observation at a given age at entry, and at all durations at entry, as recorded on the Enumerating cards for Entrants, was compared with the number of cases passing out of observation under the same age at entry, at all durations at exit, as recorded on the Enumerating cards for Emergents, these numbers being, of necessity, equal.

(44). ABSTRACT OF DATA CARDS.—Having now obtained the data in a small compass, it was necessary to bring together those portions which pertained to Select Tables, and those which pertained to Aggregate Tables. For this purpose, a distinct set of cards, designated "Abstract of Data" cards, was used. Specimens of the type of cards employed for the record of Emergents, are given in Table VIII.

(45). WITHDRAWALS.—Considering first the data for Withdrawals, these were transferred from the corresponding Enumerating cards to Abstract of Data cards of the form (*a*), Table VIII, which

included, for any class of assurance, and in respect of a particular age at entry and duration at exit, the whole of the data required for the tabulation of the withdrawals, whether in the form of Select or of Aggregate Tables. The addition of the items recorded upon the cards under the headings "S and A" and "*PD*," gave the data for Aggregate Tables, and the addition to these of those recorded under the heading "S," gave the data for Select Tables. From these cards, Schedules showing the distribution of withdrawals were entered up in the form shown in Table III (p. 45), and the fractional exposure calculated, for durations 0 to 29 inclusive, in accordance with the Modified Nearest Duration Method, as already explained in §§ (12) to (14), *supra*. A practical example of the calculation is given in the Schedule, illustrating the method and data employed for the purpose of Select Tables. For Aggregate Tables, a Schedule similar in form was employed, but with the heading "Age Attained $=[x]+t$" (where x=the Age at Assurance, and t the curtate duration at exit) instead of "Age at Assurance $=[x]$." Column (10) of the Schedules of withdrawals furnished the values of $W^{(a)}_{[x]+t}$, the number of withdrawals which are given no tabular exposure in the year of exit, and Column (9) the values of $W^{(b)}_{[x]+t}$, the number of withdrawals which are given a full year's exposure in the year of exit (*i.e.*, the number which represents the computed aggregate exposure of the whole body), for all values of x, and for values of t from 0 to 29 inclusive, both for Select and Aggregate Tables, and these numbers were entered up on Abstract of Data cards of the form (*b*), Table VIII, on the lines "Select Tables" or "Aggregate Tables." For durations 30 and upwards (which occur only in "Old" Assurances) the Nearest Duration Method, as ordinarily applied, was employed, as the numbers were relatively insignificant at these older durations; and it was found unnecessary to enter up the cases in Abstract of Data cards of the form (*a*), or in schedules of withdrawals. The sum of the numbers for W(1) and W(4) at any age at entry, and at each duration exceeding 29, were transferred directly from the Enumerating cards to Abstract of Data cards of the type (*b*), and represented the value of $W^{(a)}$, the cases of withdrawal falling in the first half of the year; whilst the sum of the numbers for W(7) and W(10), similarly transferred, represented the value of $W^{(b)}$, the cases of withdrawal falling in the second half of the year.

(46). DEATHS, AND EXISTING.—For the data in respect of Deaths and Existing, Abstract of Data cards similar in form to type (*b*), Table VIII were employed, and were entered up directly from the Enumerating cards.

(47). TERMINATIONS, ENDOWMENT ASSURANCES. — These cases were entered up directly from the Enumerating cards, upon Abstract of Data cards of the type (*b*), Table VIII, headed $T^{(a)}$ and $T^{(b)}$, according as the termination took effect (for tabular purposes) at the beginning or at the end of the policy-year current at exit. The complete data as to *Emergents*, whether by Withdrawal, Death, Existing, or Termination, were thus recorded upon cards of this type, both for Select and Aggregate Tables.

(48). ABSTRACT OF DATA CARDS, ENTRANTS.—The next step was to obtain a record, in a form convenient for tabulation, of the *Entrants*. The duration at entry under observation being as set forth in the tabular statement in paragraph (43) *supra*, it will be seen that four several types of card were required for the record of the Entrants. These are set out in Tables IX and X and include cards for the tabulation of Entrants for *Select* Tables, (*c*) "Old" Assurances (Durations 1 and upwards), (*d*) "New" Assurances (Duration 0); and cards for the tabulation of Entrants for *Aggregate* Tables, (*e*) entering under observation at a date later than that of original assurance (Durations 1 and upwards), (*f*) entering under observation from the date of original assurance (Duration 0).

(49). The above system of cards for recording the data proved to be convenient and rapid in working, as any desired combinations of data could easily be made. It was also intended that the cards should form a permanent record of the experience, in such manner that future investigators would be enabled to have access to the original facts, exhibited in a completely analytical form.

VI.—AS TO THE FORMULÆ AND METHODS ADOPTED IN DEDUCING THE NUMBERS EXPOSED TO RISK.

(50). **Select Tables.**—From the Abstract of Data cards, completed as above, the appropriate data were transferred to Working Sheets for the computation of the Numbers Exposed to Risk, the form employed for Select Tables being that given in Table XI (p. 70). The specimen there selected relates to the class of Endowment Assurances, as in the tabulation of that class, two additional columns (6) and (7) are provided for the record of Terminations; but, with this exception, the form adopted for Whole-Life Assurances was identical with the specimen here given.

(51). The formulæ and methods employed for the computation of the numbers Exposed to Risk for *Select* Tables will, it is hoped, be clearly seen upon reference to the specimen Working Sheet given

in Table XI; they are, however, here fully set out for convenient reference :—

Let $[x]$ = the tabular age at entry;

t = the tabular duration, or number of years elapsed since entry;

$\sigma_{[x]+t}$ = the ENTRANTS* coming under observation upon the tth policy anniversary dating from entry;

$\theta_{[x]+t}$ = the DEATHS, having a curtate duration of t years, that is, passing out of observation during the currency of the $(t+1)$th year of assurance;

$w_{[x]+t}$ = the WITHDRAWALS, tabulated as passing out of observation after t years from entry; that is, the sum of those at the end of the tth year of assurance, $W^{(b)}_{[x]+t-1}$, and those at the beginning of the $(t+1)$th year of assurance, $W^{(a)}_{[x]+t}$;

$T_{[x]+t}$ = the TERMINATIONS, tabulated as passing out of observation after t years from entry; that is, the sum of those at the end of the tth year of assurance, $T^{(b)}_{[x]+t-1}$, and those at the beginning of the $(t+1)$th year of assurance, $T^{(a)}_{[x]+t}$;

$\epsilon_{[x]+t}$ = the cases EXISTING, at the close of the period of observation in 1893, upon the tth policy anniversary;

$G_{[x]+t}$ = the "net movement" of Entrants and Emergents arising at duration t;

$$= \sigma_{[x]+t} - (\theta_{[x]+t-1} + w_{[x]+t} + T_{[x]+t} + \epsilon_{[x]+t});$$

and $E_{[x]+t}$ = the NUMBER EXPOSED TO RISK in the $(t+1)$th year of assurance.

Then we have

$$E_{[x]+0} = \sigma_{[x]+0} - w_{[x]+0} = G_{[x]+0} \quad \ldots\ldots\ldots\ldots \quad (1)$$

for the Number Exposed to Risk in the first year of assurance;

$$E_{[x]+t} = E_{[x]+t-1} + \sigma_{[x]+t} - (\theta_{[x]+t-1} + w_{[x]+t} + T_{[x]+t} + \epsilon_{[x]+t})$$
$$= E_{[x]+t-1} + G_{[x]+t} \quad \ldots\ldots\ldots\ldots \quad (2)$$

for the calculation of successive values by a continued method;

$$\text{and } E_{[x]+t} = \sum_{\tau=0}^{\tau=t} (\sigma_{[x]+\tau} - w_{[x]+\tau} - T_{[x]+\tau} - \epsilon_{[x]+\tau}) - \sum_{\tau=0}^{\tau=t-1} \theta_{[x]+\tau}$$
$$= \sum_{\tau=0}^{\tau=t} G_{[x]+\tau} \quad \ldots\ldots\ldots\ldots \quad (3)$$

for the verification of intermediate or final values.

* The symbol σ was employed because the corresponding cases in the Annuity Experience were, in the first instance, tabulated as "Number *Surviving* on policy anniversary."

TABLE IX.

ABSTRACT OF DATA CARDS.—ENTRANTS.—SELECT TABLES.

(c)

ASSURANCE DATA 1863-1893		M			
Assuring Age		CLASS O			
Duration		"OLD" ASSURANCES.			
Age attained		θ	ϵ	w	Total.
Entrants. *Select.*					
S. & A.	P				
	N				
	P+N				
P.D.	P				
	N				
	P+N				
S.	P				
	N				
	P+N				
Total	P				
	N				
	P+N				

NOTE.—For "OLD & NEW" Assurances the figures are the same as above.

(d)

ASSURANCE DATA 1863-1893		M			
Assuring Age		CLASS O			
Duration	0	"NEW" ASSURANCES.			
Age attained		θ	ϵ	w	Total
Entrants. *Select.*					
S. & A.	P				
	N				
	P+N				
P.D.	P				
	N				
	P+N				
S.	P				
	N				
	P+N				
Total	P				
	N				
	P+N				

NOTE.—For "OLD & NEW" Assurances the figures are the same as above.

Types (*c*) and (*d*).—Cards of this type were employed for the record of the ENTRANTS for *Select* Tables, according to the policy durations at which they entered under observation, separately tabulated as emerging by Death (θ), Existing (ϵ), or Withdrawal (w). In the case of cards of the type (*c*), "Old" Assurances, the commencing duration was that attained on the policy anniversary in 1863; in the case of cards of the type (*d*), "New" Assurances, the observations commenced with the original date of the Assurance (duration 0).

In the class of Endowment Assurances, the cases of Termination were included with the Withdrawals, the column being headed "w and τ."

TABLE X.

ABSTRACT OF DATA CARDS.—ENTRANTS.—AGGREGATE TABLES.

(*e*)

ASSURANCE DATA 1863-1893		M			
Assuring Age		CLASS O			
Duration		Entrants. *Aggregate.*			
Age attained		θ	ϵ	w	Total.
S. & H.	OLD P				
	OLD N				
	$O(P+N)$				
P.D. A	OLD P				
	OLD N				
	$O(P+N)$				
"Old" Tables	P				
	N				
	P+N				
P.D. A *"New" Tables*	P				
	N				
	P+N				
"Old" & "New" Tables	P				
	N				
	P+N				

(*f*)

ASSURANCE DATA 1863-1893		M			
Assuring Age		CLASS O			
Duration	O	Entrants. *Aggregate.*			
Age attained		θ	ϵ	w	Total.
S. & H.	NEW P				
	NEW N				
	$N(P+N)$				
P.D. A	NEW P				
	NEW N				
	$N(P+N)$				
"New" Tables	P				
	N				
	P+N				
"Old" & "New" Tables	P				
	N				
	P+N				

Types (*e*) and (*f*).—Cards of this type were employed for the record of the ENTRANTS for *Aggregate* Tables, according to the policy durations at which they entered under observation, separately tabulated as emerging by Death (θ), Existing (ϵ), or Withdrawal (w). In the class of Endowment Assurances, the Terminations were included with the Withdrawals (w and τ). Cards of the type (*e*) comprised the record of all cases entering under observation at a later policy duration than that of original entry, and included "Old" Assurances, coming under observation from the policy anniversary in 1863, and also cards in the group *PD* (Old and New Assurances) observed from the "commencing duration" recorded on the original data card. Cards of the type (*f*) comprised "New" Assurances, observed from original entry (duration o); and also cases in the *PD* group, whose "commencing duration" was o; that is, cases which, whilst overlapped at their inception by another policy on the same life, came under independent observation (on the withdrawal of the former case) within the first six months of their duration.

(52). The above formulæ represent theoretically the full processes involved in deducing the numbers exposed to risk; but, in consequence of the limitations of the data, certain terms necessarily vanish in practically applying them to the case of Old or New Assurances. Thus, in the case of Old Assurances, $\sigma_{[x]+0}$, $\theta_{[x]+0}$, and $w_{[x]+0}$ all disappear; $\epsilon_{[x]+0}$ is non-existent for both Old and New Assurances; and $T_{[x]+t}$ does not arise in the class of Whole-Life Assurances, nor, in the earlier years of assurance, in the Endowment Assurance class. These practical modifications, however, in no way affect the accuracy of the several formulæ, nor their application to any particular class or section of the experience.

(53). As a practical illustration of the employment of the formulæ above given, and of the Working Sheet (Table XI), a numerical example may be added:—

$$E_{[25]+0} = \sigma_{[25]+0} - w_{[25]+0}$$
$$= 7,141 - 125 = 7,016$$

$$E_{[25]+9} = \sum_{\tau=0}^{\tau=9} (\sigma_{[25]+\tau} - w_{[25]+\tau} - T_{[25]+\tau} - \epsilon_{[25]+\tau}) - \sum_{\tau=0}^{\tau=8} \theta_{[25]+\tau}$$
$$= 7,141 - (483 + 1,078) - 0 - 4,009 - 145$$
$$= 1,426$$

$$\left[\text{or } E_{[25]+9} = \sum_{\tau=0}^{\tau=9} G_{[25]+\tau} = 7,016 - 5,590 = 1,426\right]$$

$$E_{[25]+10} = E_{[25]+9} + G_{[25]+10}$$
$$= 1,426 - 192 = 1,234.$$

(54). **Aggregate Tables.**—The form of Working Sheet employed for the calculation of the Numbers Exposed to Risk in the construction of *Aggregate* Tables, given in Table XII, was identical in form with that employed for Select Tables, excepting only that the heading of the sheet was "Age Attained $= [x] + t$," instead of "Age at Entry $= [x]$." It follows that the entrants and emergents were scheduled on a given working sheet in respect of the constant age attained, in such manner that the data tabulated was in respect of all entry ages and durations, the sum of which made up such constant age. Thus, for example, upon the working sheet headed "Age Attained $40 = [x] + t$," the data for Aggregate Tables would include the cases coming under observation, and the cases emerging, at entry age 40 after duration 0; at entry age 39 after duration 1; and so on, up to and including the youngest entry age, in respect of which data was recorded at a duration which made up the attained age of 40. The *totals* of the

numbers recorded at all durations at the foot of each working sheet, as Entrants, Deaths, Withdrawals, Terminations, and Existing, represented the constituents of the Exposed to Risk, so far as related to cases coming under observation, or passing out of observation, at the age (40), specified at the head of the sheet; and the "net movement" of these total Entrants and Emergents represented the *increment* of the Exposed to Risk, in passing from the next lower age, to that age. By transferring, therefore, to Summary Sheets the "net movement" in respect of each age attained, and summing continuously this net movement from the earliest age, up to and including age 40, the Number Exposed to Risk at that age was deduced. (*See* Note to Table XII.)

(55). The formulæ employed for the computation of the Numbers Exposed to Risk for Aggregate Tables may be set out as follows :—

Let x = the tabular age attained, being the sum of the tabular age at entry and the tabular duration;

σ_x = the ENTRANTS, coming under observation at the tabular age x;

θ_x = the DEATHS, passing out of observation between tabular ages x and $(x+1)$;

w_x = the WITHDRAWALS, passing out of observation at the tabular age x;

T_x = the TERMINATIONS, passing out of observation at the tabular age x;

ϵ_x = the cases EXISTING, at the close of the period of observation in 1893, at the tabular age x;

each of the above functions being obtained by combining the respective cases of Entrants, or Emergents, arising at all entry ages and durations, the sum of which makes up the tabular attained age x; so that each function is of the general form

$$\phi_x = \sum_{\tau=x}^{\tau=0} \phi_{[x-\tau]+\tau}$$

Thus, for example :—

$$\epsilon_x = \epsilon_{[0]+x} + \epsilon_{[1]+x-1} + \epsilon_{[2]+x-2} + \ \ldots\ldots\ + \epsilon_{[x-1]+1} + \epsilon_{[x]+0};$$

and similarly with σ_x, w_x, T_x, and θ_x.

Also, let

G_x = the "net movement" of Entrants and Emergents at the tabular age x

$$= \sigma_x - (\theta_{x-1} + w_x + \mathrm{T}_x + \epsilon_x);$$

and E_x = the NUMBER EXPOSED TO RISK in the year following the tabular age x;

TABLE XI.

ENDOWMENT ASSURANCE EXPERIENCE 1863-1893

SELECT TABLES **MALE LIVES**

ABSTRACT OF DATA, *with computation of Numbers* EXPOSED TO RISK

CLASS **E** **Age at Entry 25=[x]** SECTION **P**

	"New" Assurances							With Profits			
			WITHDRAWALS		TERMINATIONS						
DURATION	ENTRANTS	DEATHS	FRACTIONAL EXPOSURE	BALANCE	FRACTIONAL EXPOSURE	BALANCE	EXISTING IN 1893	TOTAL DECREMENT	NET MOVEMENT		EXPOSED TO RISK
	at Exact Duration t	Curtate Duration $t-1$	$t-1$	t	$t-1$	t	at Exact Duration t	(3)+(4)+(5)+(6)+(7)+(8)	(2)−(9)		$\Sigma_0^t(10)$
		Modifications at Exact Duration t									
(t)	$\sigma_{[x]+t}$	$\theta_{[x]+t-1}$	$W^{(b)}_{[x]+t-1}$	$W^{(a)}_{[x]+t}$	$T^{(b)}_{[x]+t-1}$	$T^{(a)}_{[x]+t}$	$\epsilon_{[x]+t}$	$\theta+w+T+\epsilon$	$G_{[x]+t}$		$E_{[x]+t}$
									+	−	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		(11)
0	7,141	—	—	125	—	—	—	125	7,016	—	7,016
1	...	16	147	431	...	...	808	1,402	...	1,402	5,614
2	...	17	123	190	...	...	620	950	...	950	4,664
3	...	20	74	104	...	...	587	785	...	785	3,879
4	...	25	38	68	...	...	469	600	...	600	3,279
5	...	13	22	56	...	...	420	511	...	511	2,768
6	...	15	30	34	...	...	356	435	...	435	2,333
7	...	20	19	31	...	...	262	332	...	332	2,001
8	...	14	19	22	...	...	277	332	...	332	1,669
9	...	5	11	17	...	...	210	243	...	243	1,426
10	...	7	5	10	...	...	170	192	...	192	1,234
11	...	4	10	16	...	...	158	188	...	188	1,046
12	...	4	7	13	...	...	132	156	...	156	890
13	...	6	8	2	...	...	129	145	...	145	745
14	...	6	2	7	...	...	80	95	...	95	650
15	...	4	3	5	1	1	79	93	...	93	557
16	...	7	4	7	...	...	77	95	...	95	462
17	...	3	2	2	...	...	71	78	...	78	384
18	...	2	1	1	...	...	68	72	...	72	312
19	...	3	...	1	...	...	46	50	...	50	262
20	...	...	1	...	...	...	39	40	...	40	222
21	...	2	...	1	1	...	22	26	...	26	196
22	...	3	1	...	...	...	21	25	...	25	171
23	...	1	...	...	...	...	20	21	...	21	150
24	...	3	1	1	...	2	30	37	...	37	113
25	...	2	1	1	5	13	20	42	...	42	71
26	...	1	1	...	3	...	18	23	...	23	48
27	...	...	...	...	...	...	12	12	...	12	36
28	...	1	...	...	...	...	17	18	...	18	18
29	...	..	...	1	...	...	7	8	...	8	10
30	...	...	...	...	1	...	9	10	...	10	...
	7,141	204	530	1,146	11	16	5,234	7,141	7,016	7,016	42,226

TABLE XII.

ENDOWMENT ASSURANCE EXPERIENCE, 1863-1893

AGGREGATE TABLES — MALE LIVES

ABSTRACT OF DATA, *with computation of Numbers* EXPOSED TO RISK

CLASS **E** — **Age 40 − [x] + t** — SECTION **A P**

"New" Assurances — With Profits

Duration	Entrants	Deaths	Withdrawals		Terminations		Existing in 1893	Total Decrement	Net Movement		Exposed to Risk
			Fractional Exposure	Balance	Fractional Exposure	Balance					
	at Exact Duration t	Curtate Duration t−1	t−1	t	t−1	t	at Exact Duration t	(3)+(4)+(5)+(6)+(7)+(8)	(2)−(9)		$\Sigma_0^t(10)$
		Modifications at Exact Duration t									
(t)	$\sigma_{[x]+t}$	$\theta_{[x]+t-1}$	$W^{(b)}_{[x]+t-1}$	$W^{(a)}_{[x]+t}$	$T^{(b)}_{[x]+t-1}$	$T^{(a)}_{[x]+t}$	$\epsilon_{[x]+t}$	$\theta+w+T+\epsilon$	$G_{[x]+t}$ +	−	$E_{[x]+t}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)		(11)
0	1,915	...	...	17	...	...	...	17	1,898	...	*1,898*
1	2	4	22	77	...	...	217	320	...	318	*1,557*
2	2	11	27	38	...	...	213	289	...	287	*1,524*
3	2	8	14	29	...	...	209	260	...	258	*1,341*
4	1	8	11	23	...	...	192	234	...	233	*1,327*
5	1	9	10	26	...	2	222	269	...	268	*1,333*
6	1	9	10	20	...	...	210	249	...	248	*1,250*
7	1	14	9	12	...	...	179	214	...	213	*1,278*
8	1	11	5	12	...	...	171	199	...	198	*1,113*
9	1	9	7	11	...	1	147	175	...	174	*1,025*
10	...	9	9	11	...	2	136	167	...	167	*944*
0–5	*1,923*	*40*	*84*	*210*	...	*2*	*1,053*	...	...	...	*7,647*
0–10	*1,927*	*92*	*124*	*276*	...	*5*	*1,896*	...	...	...	*13,646*
11	2	5	6	12	1	...	125	149	...	147	...
12	...	4	5	5	1	1	124	140	...	140	...
13	...	5	3	6	...	1	107	122	...	122	...
14	...	6	3	8	...	1	96	114	...	114	...
15	...	4	3	5	1	1	77	91	...	91	...
16	...	3	2	1	...	...	89	95	...	95	...
17	...	4	4	3	...	2	51	64	...	64	...
18	...	2	2	1	...	...	52	57	...	57	...
19	...	...	...	2	1	1	28	32	...	32	...
20	...	2	1	...	1	...	10	14	...	14	...
21	...	1	...	1	...	...	10	12	...	12	...
22	...	2	...	...	...	...	6	8	...	8	...
23	...	...	...	...	...	...	1	1	...	1	...
24	...	...	...	...	...	...	3	3	...	3	...
25	...	...	...	...	...	...	2	2	...	2	...
26	...	...	...	...	...	...	1	1	...	1	...
27	...	...	...	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...	...	...	...
30	...	...	...	...	...	...	...	...	...	...	...
	1,929	130	153	320	5	12	2,678	3,298	1,898	3,267	...
									= −1,369		

NOTE.—In the construction of *full* aggregate tables, the total line on each Working sheet was transferred to Summary sheets of the same form, which supplied the aggregate data as to entrants, deaths, &c., in respect of each age attained; whilst the continued summation of the "net movement" (col. 10) from the earliest age, up to and including any given age, supplied the number exposed to risk at that age attained.

The figures printed in *italics* supplied the data for the construction of *Truncated* aggregate tables. The Exposed to risk for duration t were deduced by adding, to the Exposed for duration $(t-1)$ in the *preceding* Working sheet, the "net movement" for duration t in the same Working sheet. Thus $E_{[35]+4}+G_{[35]+5}=E_{[35]+5}$, or 1,601 − 268 = 1,333. This operation was performed on cards, as explained in §§ (61) (62), of the text.

Then we have

$$E_x = E_{x-1} + \sigma_x - (\theta_{x-1} + w_x + T_x + \epsilon_x)$$
$$= E_{x-1} + G_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

for the computation of successive values by a continued method; and

$$E_x = \sum_{a=0}^{a=x} (\sigma_a - w_a - T_a - \epsilon_a) - \sum_{a=0}^{a=x-1} \theta_a$$
$$= \sum_{a=0}^{a=x} G_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

for verification of intermediate and final values.

(56). **Truncated Aggregate Tables.**—It will be convenient to state, in the first place, the theoretical formulas for the computation of the entrants and emergents, and the numbers exposed to risk, in respect of the "*Truncated*" aggregate Tables, from which are excluded the data for certain of the early years of assurance. The practical application of these formulas in the present experience can then be explained, and illustrated by examples.

(57). The Entrants, at attained age x, in respect of the truncated Table excluding the first t years from date of assurance, comprise (i) *Original Entrants*, coming under first observation at age x, after durations of $(t+1)$ years and upwards; (ii) *Surviving Entrants*, effected at age at entry $[x-t]$, coming under original observation after durations 0 to t years inclusive, and completing t years' duration at age x; the cases in both classes being deduced from the analysed data for aggregate Tables. The cases in class (i) are equal to

$$\sum_{\tau=t+1}^{\tau=x} \sigma_{[x-\tau]+\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

whilst those in class (ii) can evidently be derived from the number exposed to risk in respect of entry age $[x-t]$, after t years' duration, by the addition of the withdrawals and terminations which, in the computation of the numbers exposed to risk, are treated as of tabular duration t, although actually emerging in the earlier portion of the $(t+1)$th year of assurance. The formula for the Surviving Entrants of class (ii) is therefore

$$E_{[x-t]+t} + W^{(a)}_{[x-t]+t} + T^{(a)}_{[x-t]+t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The total entrants of classes (i) and (ii), represented by the sum of the expressions in formulas (6) and (7), may conveniently be designated $\sigma^{(t)}_x$.

(58). Dealing now with the Emergents of different classes, and designating the Withdrawals, Terminations, Existing, and Deaths,

entering into the construction of the numbers exposed to risk at age x in the t years' truncated Table, by $w_x^{(t)}$, $T_x^{(t)}$, $\epsilon_x^{(t)}$, and $\theta_{x-1}^{(t)}$ respectively, we have the following formulas for the several classes of emergents:—

$$w_x^{(t)} = W_{[x-t]+t}^{(a)} + \sum_{\tau=t+1}^{\tau=x} w_{[x-\tau]+\tau}$$

$$T_x^{(t)} = T_{[x-t]+t}^{(a)} + \sum_{\tau=t+1}^{\tau=x} T_{[x-\tau]+\tau}$$

$$\epsilon_x^{(t)} = \sum_{\tau=t+1}^{\tau=x} \epsilon_{[x-\tau]+\tau}$$

$$\theta_{x-1}^{(t)} = \sum_{\tau=t+1}^{\tau=x} \theta_{[x-\tau]+\tau-1}$$

the cases being throughout taken from the analysed data for aggregate Tables.

(59). The "net movement" of emergents and entrants at age x, which may be designated $G_x^{(t)}$, is thus

$$G_x^{(t)} = \sigma_x^{(t)} - (w_x^{(t)} + T_x^{(t)} + \epsilon_x^{(t)} + \theta_{x-1}^{(t)}) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the formula for deducing the number exposed to risk at successive ages by a continued method is

$$E_x^{(t)} = E_{x-1}^{(t)} + G_x^{(t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and, for verification of intermediate and final values,

$$E_x^{(t)} = \sum_{a=t}^{a=x} G_a^{(t)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

(60). An alternative formula for deducing the numbers exposed to risk in the truncated Table may be obtained by deducting, from the total number exposed to risk at age x in the *full* Aggregate Table, the number exposed in respect of entry ages $[x]$ to $[x-t+1]$ inclusive, after durations of 0 to $(t-1)$ years, respectively; so that the number exposed to risk in the t years' truncated Table is equal to

$$E_x^{(t)} = E_x - (E_{[x]+0} + E_{[x-1]+1} + \quad . \quad . \quad . \quad + E_{[x-t+1]+t-1}) \quad . \quad . \quad . \quad (11)$$

where the values of E are throughout deduced from the data for aggregate Tables.

(61). Formula (11) was that practically employed in the computation of the numbers exposed to risk for the truncated aggregate Tables. For this purpose, the data for the first t years had to be arranged in the *form* of select tables (discriminating the entry ages and durations) in order to ascertain the quantity to be deducted from the number exposed to risk at any age in the full aggregate Table, to arrive at that for the truncated Table. This was practically given effect to by transferring the "net movement,"

$G_{[x]+\tau}$, which forms the basis of the number exposed to risk, to suitable cards, each of which recorded the net movement in respect of a particular value of $[x]$, and of successive values of τ from 0 to 10 inclusive. The continued summation of the values of G, thus recorded, gave the successive values of $E_{[x]+0}$, $E_{[x]+1}$, &c. (*see* formula 3, p. 65); and the numbers exposed to risk, thus deduced, were then re-arranged according to ages attained, in the form

$$E_{[x]+0}+E_{[x-1]+1}+ \quad . \quad . \quad . \quad +E_{[x-t+1]+t-1}$$

and their sum was deducted from the value of E_x, the full aggregate exposure at age x.

(62). The methods followed may be illustrated by a numerical example from the Endowment Assurance experience. Taking age 40 at date of assurance, the value of $G_{[40]+\tau}$, the net movement contributing to the number exposed to risk, was extracted, for all values of τ from 0 to 10 inclusive, from the Working Sheets for Aggregate Tables (*see* Table XII), headed (on successive pages) with

TABLE XIII.

"TRUNCATED" AGGREGATE TABLES.				M
Class E.		Section ℑP.		
Age at Assurance 40.				
Duration.	*Age attained.*	*Net Movement.* +	−	*Exposed to Risk.*
0.	40	1,898	–	1,898
1.	41	–	329	1,569
2.	42	–	273	1,296
3.	43	–	236	1,060
4.	44	–	174	886
5.	45	–	160	726
6.	46	–	116	610
7.	47	–	95	515
8.	48	–	92	423
9.	49	–	71	352
10.	50	–	55	297*

* The value of $E_{[40]+10}$ is not required for the purpose of deducing the exposures under the ten years' truncated Table, by the method of formula (11); but this value enters into the construction of the "surviving entrants" (formula 7), and is therefore required where the exposures in the truncated Table are deduced from the entrants and emergents at each age.

the attained ages 40, 41, ... 50. The values of the net movement were conveniently entered upon a special form of card (*see* Table XIII), headed "Age at Assurance 40," and the continued summation, on the card, of the net movement, gave the number exposed to risk, in respect of cases, originally entering at age 40, which came under observation, or passed out of observation, during the first eleven* years of assurance.

(63). The numbers exposed to risk, thus deduced, were then conveniently entered in column (11) of successive pages of the Working Sheets for Aggregate Tables (Table XII), under their appropriate ages attained. The numbers entered in this column supplied at once the material for obtaining, at each age, the number to be deducted from the total number exposed to risk, E_x, in order to obtain the number exposed for the truncated Table, $E_x^{(t)}$, for all values of t from 1 to 10 inclusive. For example, in order to deduce the value of $E_{40}^{(5)}$,—the number exposed to risk in the truncated aggregate Table, after excluding the experience of the first five years,—all that was necessary was to deduct, from the number exposed in the full aggregate Table, E_{40}, the sum of the first five entries in column (11) of the Working Sheet for age attained 40. Thus, the full number exposed to risk at age 40, in the class of New Participating Endowment Assurances, were 19,443 in number. The first five entries, under age 40, column (11) of the Working Sheet (Table XII) make up a total of 7,647; and deducting this number from the full number exposed to risk, we arrive at 11,796, as the value of $E_{40}^{(5)}$.

(64). Proceeding similarly with the data in each of the four sections constituting the Endowment Assurance Experience, we obtain the following results for attained age 40:—

TABLE XIV.

ENDOWMENT ASSURANCE EXPERIENCE.

ATTAINED AGE, 40.

SECTION OF EXPERIENCE.	NUMBER EXPOSED TO RISK.		
	Full Table.	First Five Years.	Truncated Table.
New—Participating . .	19,443	7,647	11,796
„ Non-Participating .	7,373	2,310	5,063
Old—Participating . .	1,394	79	1,315
„ Non-Participating .	1,170	91	1,079
Combined Experience . .	29,380	10,127	19,253

* See foot-note on previous page.

The numbers exposed to risk in the full Table, and in the Truncated Table, for the Combined Experience, agree with those given, at age 40, in the Tables on pages 136 and 137 of the published volume of Unadjusted Data (Endowment Assurances and Minor Classes of Assurance).

(65). The Entrants and Emergents at each age for the truncated Tables, as set out in the above volume, were deduced from the analysed data for full aggregate Tables, by the formulas given in §§ (57) and (58) above. The numbers exposed to risk, as previously deduced at each age by the methods explained and illustrated in §§ (60) to (64), were then independently obtained (for purposes of verification) by deducing, from the entrants and emergents in the truncated Table, the "net movement" at each age (formula 8), and then computing the numbers exposed to risk, by continued summation of the net movement (formulas 9, 10).

(66). These processes can be illustrated by an example taken from the data, in respect of the 5 years' truncated Table, given on page 137 of the published volume of Unadjusted Data:—

$$E^{(5)}_{40} = E^{(5)}_{39} + G^{(5)}_{40}$$
$$= E^{(5)}_{39} + \sigma^{(5)}_{40} - (w^{(5)}_{40} + T^{(5)}_{40} + \epsilon^{(5)}_{40} + \theta^{(5)}_{39})$$
$$= 19{,}657 + 2{,}108 - (315 + 43 + 2{,}016 + 138) = 19{,}253$$

Also
$$E^{(5)}_{40} = \sum_{\alpha=5}^{\alpha=40} G^{(5)}_{\alpha}$$
$$= \sum_{\alpha=5}^{\alpha=40} [\sigma^{(5)}_{\alpha} - (w^{(5)}_{\alpha} + T^{(5)}_{\alpha} + \epsilon^{(5)}_{\alpha})] - \sum_{\alpha=5}^{\alpha=39} \theta_{\alpha}$$
$$= 47{,}754 - (4{,}787 + 115 + 22{,}339 + 1{,}260) = 19{,}253.$$

(67). Truncated Aggregate Tables are included in the published volume of Unadjusted Data, in respect of the undermentioned classes of assurance:—

Class.	Years of Assurance Excluded.
Males—	
Whole-Life Participating ...	5, 6, 7, 8, 9 and 10 years
,, ,, Non-participating ...	5, 6, 7, 8, 9 and 10 years
,, ,, Limited Payments ...	5 and 10 years.
,, ,, Ascending Scale ...	5 and 10 years.
Endowment Assurances... ...	5 and 10 years.
Females—	
Whole-Life Participating ...	5 and 10 years.
,, ,, Non-Participating ...	5 and 10 years.

VII. AS TO THE METHODS FOLLOWED IN CLASSIFYING AND TABULATING THE DATA FOR THE MINOR CLASSES OF ASSURANCE.

(68). It remains only to describe the methods adopted in the case of the Minor Classes of Assurance, which differed somewhat from those described above, owing partly to the fact that duplicates were only eliminated as arising at the same age at assurance (in other words, that the data for Select and Aggregate Tables were identical), and partly to the smaller extent of tabulation necessary. For the following classes of the Female Experience, where the numbers were too small to render any further tabulation of value, summaries of data only were prepared, viz.:—Whole-Life Assurances with Ascending Premiums; Whole-Life Assurances with Limited Premiums; Contingent Assurances; and Temporary Assurances.

(69). **Contingent Assurances, Male Lives—Temporary Assurances, Male Lives.**—As the assurances in these classes were usually of short duration, it was considered sufficient to tabulate the data in the form of Select Tables extending over the first ten years of assurance, with a summary statement of the data in respect of longer durations. As the numbers under observation at the individual ages at entry were not considerable, they were throughout tabulated at grouped entry ages 0–17, 18–22, 23–27, &c., the final groups being 63–77 for Contingent Assurances, and 63–89 for Temporary Assurances. Cards of a special form, which served the purpose of both Enumerating and Abstract of Data cards—*see* Appendix R—were employed for the record of the Entrants and Emergents in these classes. These assurances being almost invariably without profits, no discrimination was made between participating and non-participating assurances. The data for "Old" and "New" Assurances were added together before being employed in the processes of tabulation, but the separate particulars were recorded on the cards, in case they should be required at any future time. With these exceptions, the methods of sorting and tabulation adopted were similar to those followed in the main classes of Whole-Life and Endowment Assurances, which have been already described in detail.

(70). **Whole-Life Assurances with Limited Number of Premiums, Male Lives; Whole Life Assurances with Ascending Scale of Premiums, Male Lives; Joint-Life Assurances, Male Lives; Joint-Life Assurances, Female Lives.**—In these classes, the data for which were much more extensive,

and over longer durations than those previously adverted to, it was decided to tabulate the data in the form of Select Tables for the first ten years of assurance (with a summary statement of the subsequent data), and also in the alternative form of Aggregate Tables. For the Select Tables, processes were followed identical with those described above for Contingent and Temporary Assurances, with the exceptions (i) that in the case of the Whole-Life Assurances with Limited or Ascending Premiums the Select Tables were prepared for each age at entry; (ii) that the data for "Old" Assurances, with profits, "Old" Assurances, without profits, "New" Assurances, with profits, and "New" Assurances, without profits, were separately entered on the cards, and combined for purposes of tabulation. In the case of Joint-Life Assurances, the ages at entry were grouped for Select Tables, as in the case of Contingent and Temporary Assurances. The special form of card given in Appendix R (*a*) was also employed for the record of the Entrants and Emergents in these classes.

(71). The methods adopted in constructing the Aggregate Tables were similar to those already described for the main classes of assurance. A special form of card—*see* Appendix R (*b*)—was employed in these classes for the record of the data for Aggregate Tables. For the classes of Whole-Life Assurances with Limited and Ascending Premiums, Truncated Aggregate Tables were also constructed, eliminating the first five, and the first ten years' experience. Since the data for Select and Aggregate Tables were throughout identical, the data for these truncated Tables were readily obtained, by deducting from the numbers exposed to risk, and the deaths, at each age attained in the full Aggregate Table, the numbers exposed, and the deaths, as set out in the Select Tables, for the several years to be excluded from observation, arranged under the age attained.

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(72). In Appendix S are given some notes as to the data referred back to the contributing offices for examination and correction, the nature of the queries thus arising, and the way in which they were respectively dealt with for the purposes of the experience.

THOMAS G. ACKLAND,

Hon. Official Supervisor.

ASSURANCE EXPERIENCE.

APPENDICES

TO NOTES AS TO THE PRINCIPLES AND METHODS ADOPTED FOR CLASSIFYING AND TABULATING THE DATA.

Appendix A.

NEW COLLECTIVE MORTALITY EXPERIENCE

IN PREPARATION BY THE INSTITUTE OF ACTUARIES AND THE FACULTY OF ACTUARIES.

MEMORANDUM

FOR THE GUIDANCE OF THE COMPANIES IN FILLING IN THE MORTALITY EXPERIENCE CARDS.

ASSURED LIVES.

1. IT is intended that the New Collective Mortality Experience shall include (*a*) policies existing on the books of the Companies on the anniversaries in 1863 of the dates of entry, and (*b*) policies issued between 1st January 1863 and the 31st December 1892. A form of card will be supplied for each of these separate classes, and specimens are enclosed herewith. That headed "Old Policies" is intended for class (*a*) above named, and that headed "New Policies" for class (*b*). A card should be written for each policy that comes within the limits of the experience.

2. The male lives are to be distinguished from the female; and for the male lives white cards are to be used, and for the female, pink cards.

3. Only those policies are to be included which are on lives resident in the United Kingdom at the date of entry. Thus, cards are not to be written for policies issued through agencies abroad, or for policies issued in the United Kingdom on lives residing abroad at the date of entry.

4. All policies granted at an extra premium on impaired lives, or at an extra premium on account of occupation, or at an extra annual premium for foreign residence or for whole-world license, are to be excluded, as also all policies on the lives of Naval, Military, or Seafaring men. Policies on which a single extra premium for whole-world license has been paid are to be included. In the case of female lives, where an extra is charged only on account of sex, or only for the risk of pregnancy, such policies are to be included, and cards are to be written for them.

5. These regulations as to extra premium for foreign residence apply only where the extra premium runs from the date of issue of the policy. If after the issue of the policy the life goes abroad and subjects himself

to extra premium, no account is to be taken of the fact, and a card is to be written as if no extra risk had been incurred.

N.B.—It is suggested that those Companies which can do so should under "Remarks" record the fact that the life has gone abroad and subjected himself to extra premium, giving the date of such event. If the event, however, happened more than once, only the first date need be given.

6. Lives are not to be included which have been admitted without medical examination by virtue of special schemes or arrangements introduced since 1863; and lives of members of the Royal Family are also to be excluded.

7. Only direct policies of the Company are to be included; and therefore cards are not to be written for re-assurances *received from* other Offices.

8. It will be noticed that two descriptions of type appear on the cards. It is intended that the Companies shall fill in the particulars required only under the large capital type, and that those asked for in the small Roman type shall be filled in by the Institute and Faculty of Actuaries.

9. Taking in order the lines in capital type upon the cards, the following explanations may be useful.

10. **No.**..... The Policy No. should be inserted here.

11. £..... The amount here required is the orignal sum assured by the policy to the nearest £. If the original policy has been exchanged for one of larger amount, it should be treated as having been surrendered and a card written for it accordingly, and a new card should be written for the new policy, giving particulars as at the date of exchange. If the original assurance be continued but for a smaller amount, whether under the original or under a new policy, then the particulars of the alteration should be given at the foot of the card under the heading "Remarks."

12. **Class.** It is proposed to investigate separately some of the larger classes of assurances, such as the Whole Life Assurances, Endowment Assurances, &c., and therefore it is requested that care be taken in distinguishing the class.

The different classes of assurances to be included are as under, and should be distinguished by letters written or printed on this line as follows:—

Ordinary Whole-Life Assurances granted at Uniform Premiums payable throughout life	O.
Whole-Life Assurances granted by Single Premiums or Premiums limited in number	O. L.
Whole-Life Assurances granted by Premiums on an ascending scale, such as Assurances at half-premium rates	O. A.

Ordinary Endowment Assurances	E.
Term Assurances on Single Lives	T.
Joint Life Assurances (Whole Term)	J. L.

N.B.—A separate card should be written for each of the lives, and a reference should be made under "Remarks" to the other life or lives, thus:—"Jointly with A. B.", the date of birth of A. B. being also given.

Contingent Survivorship Assurances granted by Annual Premiums, on a single life against another life or a combination of lives	C.

N.B.—A card should be written for the assured life, but not for the counter-life or counter-lives.

All other assurances should be omitted and cards should not be written for them.

13. **"Profit or Not."** The letter "P" should be inserted in the case of a With-profit policy, and the letter "N" in the case of a Non-profit policy.

All assurances, the bonuses on which depend on the profits of the Company, or which carry guaranteed bonuses, should be included among those With-profit. Thus, it is immaterial for this purpose whether the profits are taken as a reversion, or in cash, or in reduction of premium, or whether the bonus be immediate or deferred; and also it is immaterial if the policy be issued under such conditions as go by the various names "Minimum premium", "Cost price", "Prime cost", &c. All such policies should be included in the With-profit group, and the Companies are invited to write in such cases under "Remarks" the words "Minimum premium", "Cost price", &c., as the case may be. They are also invited to add under "Remarks" the words "Part Credit", where at the commencement any portion of the premium was allowed to remain as a debt on the policy.

14. **"Life."** It is desirable that the full Surname and first Christian name, and the initials of other Christian names of the Life should be given.

The Surname should be placed on the first line, and the Christian name or names on the second. In the case of a compound Surname, such as John Brown-Smith, the last name only should be treated as Surname, the remainder being treated as part of the Christian name and given in full after the Christian name on the second line: thus

Smith,
John Brown.

Similarly in the case of a Surname with such a prefix as "de", "von", "van", "van der", &c., *e.g.*, "Van Tromp", only the Surname itself, *e.g.*, "Tromp", should be placed on the first line, and the prefix, "Van", &c., should be placed on the second line after the Christian

names. If, however, the prefix is actually incorporated in the name, *e.g.*, Vanderbilt, then the whole should appear on the first line as Surname.

In the case of a female who has changed her name by marriage, the name under which she assured should be given on these two lines, and her maiden name, or her married name as the case may be, should if possible be given under "Remarks."

In the case of a peer the family name and the Christian names should be given on these two lines, and the title should be given under "Remarks."

15. **"Date of Birth."** The date of birth should be given with as much accuracy as possible. The day of the month should be inserted under the letter "D" in the column before the hyphen, and the number of the month, thus 7 for July, should be inserted after the hyphen, the year being given in the ruled column on the right, under the word "year." If the exact date of birth cannot be given, such particulars as are possible should be supplied in this line.

16. **"Date of Entry."** Here should be written the date when the risk was assumed to commence, and not necessarily the date of the policy.

17. **"Date of Exit."** This line is intended for the date of cessation of the risk, and the exact date if possible should be supplied. Should the exact date not be forthcoming, the nearest approach to it possible should be given. Where a discontinued policy has been kept in force for a period through a special regulation of the office, such as the Non-forfeiture regulation, the date of exit in all such cases should be considered as that on which the office finally ceased to be on the risk. If a policy has been discontinued and revived for its original amount, no attention should be paid to the fact, but the policy should be treated as if there had been no break in the continuity of the risk.

18. **"Mode of Exit."** The letter "D", to be placed within the brackets, will mean that the exit was caused by death; the letter "W" by withdrawal, that is, surrender or lapse; and the letter "T" by termination in any other way, such as in the case of an Endowment Assurance, the survival of the life to the stipulated age; in the case of a term assurance, the term for which the policy was granted having expired; in the case of a contingent assurance, the death of the counter life; and in the case of a joint life assurance, the failure of the other joint life. It is unnecessary to distinguish these particular cases in the cards, and the one letter "T" will represent them all.

19. The observations are to close with the anniversary of the policy in 1893. If the policy still remained in force at its anniversary in 1893, the lines Date of Exit and Mode of Exit should be left blank; and it will be assumed in all cases where no mark is made on them, that the policy was still running at the close of the observations.

20. For the sake of distinguishing the different Companies, so that the cards may be returned after they have been used for the Mortality Experience, each Company should have a distinguishing number or letter, to be approved by the Institute and the Faculty of Actuaries, printed at the foot of the card. The Company for its own purposes may make such remarks on the back of the card as may be thought desirable; but it is particularly requested that no marks except those above mentioned be made on the face of the card.

21. The writing and figures on the cards should be made as distinct as possible, and the figures should be ranged under each other so that there may be no difficulty in reading them; and in the cards for old policies, under the heading "year", the figures should be ranged above and below those for 1863 which are printed, so that in subsequently dealing with the cards there may be no difficulty in making additions or subtractions.

22. The cards of each Company, when all completed, should be sent in to the Institute of Actuaries or to the Faculty of Actuaries arranged in any order that the Company may find convenient.

23 If any further explanations be required they will be supplied on application to the Honorary Secretaries of the Institute of Actuaries, Staple Inn Hall, Holborn, London, W.C., or to the Honorary Secretary of the Faculty of Actuaries, Edinburgh.

May 1st, 1894.

Appendix B.

NEW COLLECTIVE MORTALITY EXPERIENCE.

LIST OF ENQUIRIES made by the English Companies respecting the filling-in of the Experience Cards, and Answers given thereto.

QUESTION	ANSWER
1. How should a Policy be treated which has been transferred from one class of Assurance to another, or from "With Profit" to "Without Profit," or *vice versa*, with or without variations in the Sum Assured and Premium?	1. The Policy is to be treated as if it had remained in its original class, the date and nature of the subsequent alterations being stated under "Remarks."
2. If the original Policy has been converted into a "Paid-up" Assurance (whether a new Policy be issued or not), is a new card to be written?	2. No, the particulars of the alterations should be given at the foot of the card under "Remarks" (*vide* Clause 11 of the Memorandum).
3. Where the age next birthday only is ascertainable, how is the "date of birth" to be estimated?	3. The date of birth should be left blank, and the age next birthday stated under "Remarks."
4. Are Assurances with Premiums on a decreasing scale, and Endowment Assurances on Joint Lives, to be excluded?	4. Yes.
5. Are the Assurances on the Survivor of any number of Lives to be included?	5. No.
6. Are Lives resident in the United Kingdom at the date of entry, but known to be about to proceed and to reside abroad, to be excluded?	6. No, a card should be written for the Policy, and particulars of the extra risk and extra Premium (if any) given under "Remarks" (*vide* note to Clause 5).
7. Will the fact that a Life has been in the Naval, Military, or Merchant Service, exclude him from the experience?	7. No, not if he had retired or gone into the Reserves at the time the Policy was effected.
8. Does Military Service include service in the Militia or Volunteers?	8. No.
9. Does the expression in the note to Clause 5, "If the event happen more than once," apply to climate risks generally?	9. Yes.
10. In the case of a Lapsed Policy, is the exact date of cessation of the risk to be the date when the days of grace expired?	10. Yes, if the Company's risk actually ceased on such date, but not if it continued at risk under some Non-Forfeiture Regulation or otherwise (*vide* Clause 17).
11. Are Policies which are not renewed on their anniversary in 1893 to be treated as remaining in force on that day; for instance, a Policy effected in October, 1870, and discontinued in October, 1893?	11. Yes, if the Policy was in force up to such anniversary.
12. What is the exact interpretation to be given to the phrase "Date when the risk was assumed to commence"?	12. The "date of entry" should be the date from which the first Premium runs.
13. If a Policy may be reinstated during the lifetime of the Life Assured, within twelve calendar months after the expiration of the days of grace, without proof of health, the usual Non-Forfeiture Regulation also applying, what date should be put down as "the date of cessation of risk"?	13. As the Company is not necessarily fully on the risk during the twelve months, and might not be liable to pay the Sum Assured in the event of the death of the Life Assured before reinstatement, the date of cessation of risk should be the day when the full Sum Assured ceased to be protected by the special Non-Forfeiture Regulation (*vide* Clause 17).

Dated 4th April, 1895.

Appendix C.

CLASSIFICATION OF WHOLE-LIFE ASSURANCES AND ENDOWMENT ASSURANCES (ENGLISH AND SCOTTISH, MALE AND FEMALE) according to the interval (in Months) between the Date of Entry, and the last preceding Birthday; based upon an examination of the cases arising in decennial years of birth, and reduced to a total of 10,000 cases in each Section.

Interval (in Months) between Date of Entry and last preceding Birthday.	MALE LIVES.				FEMALE LIVES.				Interval (in Months) between Date of Entry and last preceding Birthday.
	WHOLE-LIFE ASSURANCES.		ENDOWMENT ASSURANCES.		WHOLE-LIFE ASSURANCES.		ENDOWMENT ASSURANCES.		
	English.	Scottish.	English.	Scottish.	English.	Scottish.	English.	Scottish.	
0 to 1	567	249	379	159	534	271	339	179	0 to 1
1 ,, 2	533	446	380	369	694	487	458	403	1 ,, 2
2 ,, 3	595	594	453	523	676	674	621	564	2 ,, 3
3 ,, 4	611	654	571	571	675	715	610	591	3 ,, 4
4 ,, 5	668	687	659	730	754	709	733	779	4 ,, 5
5 ,, 6	673	729	716	636	772	843	728	631	5 ,, 6
	3,647	3,359	3,158	2,988	4,105	3,699	3,489	3,147	
6 ,, 7	709	723	677	767	764	820	696	703	6 ,, 7
7 ,, 8	715	688	717	716	738	750	710	725	7 ,, 8
8 ,, 9	758	753	729	716	808	791	753	743	8 ,, 9
9 ,, 10	798	802	824	838	808	846	773	887	9 ,, 10
10 ,, 11	892	941	913	984	906	919	844	986	10 ,, 11
11 ,, 12	2,481	2,734	2,982	2,991	1,871	2,175	2,735	2,809	11 ,, 12
	6,353	6,641	6,842	7,012	5,895	6,301	6,511	6,853	
	10,000	10,000	10,000	10,000	10,000	10,000	10,000	10,000	
Average Interval... ...	7·332 mos.	7·687 mos.	7·847 mos.	7·985 mos.	6·908 mos.	7·326 mos.	7·575 mos.	7·855 mos.	Average Interval.
MEAN AGE METHOD:— (Assumed Interval 6 mos.) Deviation (−) ...	1·332 ,, (41 days)	1·687 ,, (51 days)	1·847 ,, (56 days)	1·985 ,, (60 days)	·908 ,, (28 days)	1·326 ,, (40 days)	1·575 ,, (48 days)	1·855 ,, (57 days)	MEAN AGE METHOD:— (Assumed Interval 6 mos.) (−) Deviation
NEAREST AGE METHOD:— Average Interval ...	7·624 mos.	7·968 mos.	8·208 mos.	8·412 mos.	7·074 mos.	7·560 mos.	7·812 mos.	8·220 mos.	NEAREST AGE METHOD:— Average Interval
Deviation (+) ...	·292 ,, (8⅞ days)	·281 ,, (8½ days)	·361 ,, (11 days)	·427 ,, (13 days)	·166 ,, (5 days)	·234 ,, (7⅛ days)	·237 ,, (7¼ days)	·365 ,, (11 days)	(+) Deviation

Appendix D.

RULES FOR OBTAINING THE NEAREST AGE AT ENTRY BY MODIFICATION OF THE YEAR OF BIRTH.

(1) Modification to be applied to the Year of Birth recorded on the cards in the following cases :—

(*a*) Where Day and Month of Birth *precede* Day and Month of Entry by *more than six months*, mark the Year of Birth (—)

(*b*) Where Day and Month of Birth *follow* Day and Month of Entry by *more than six months*, mark the Year of Birth (+)

(*c*) Where the interval between Day and Month of Birth and Day and Month of Entry is *exactly six months* :—

(i) If the Day and Month of Birth *precede* the Day and Month of Entry, mark the Year of Birth, in *one-half* of the cases, (—)

(ii) If the Day and Month of Birth *follow* the Day and Month of Entry, mark the Year of Birth, in *one-half* of the cases, (+)

(2) In all cases to obtain the Nearest Age at Entry deduct the Year of Birth (modified ±1 as marked in the above cases) from the Year of Entry.

Appendix E.

METHODS OF DEALING WITH CASES OF DEFECTIVE DATA AS TO BIRTH.

The cards upon which the dates of Birth were not fully recorded were set aside, and were of the following four types :—

(1) Those on which the "*Office Age at Entry*" was given (under the heading "Remarks") with no direct information whatever as to the date of birth;

(2) Those on which the *Year* only of birth was given, without the Age at Entry;

(3) Those on which both the *Year* of birth and the Office age at entry were given;

(4) Those on which the date of *Baptism* was given in lieu of the date of birth.

The number of cards so set aside was, in the Male Experience, 30,210, of which 27,470, or about 90 per cent., were Whole-Life Assurances; and, in the Female Experience, 4,753, of which 3,793, or about 80 per cent., were Whole-Life Assurances.

(1) In the first variety, "Office Age at Entry only given," the cards representing policies on the same life, effected at the same age at entry, were first brought together, in order to avoid inconsistent dealings in the later stages. In many cases the comparison of these cards on the same life indicated with certainty the nearest age at date of assurance; but where the cases could not be settled, the following method of procedure was adopted:—An examination of the tabulated statistics, as to the interval subsisting in the main body between the date of assurance and the previous birthday (*see* Appendix C) was made, and it was found that in the English Section of Whole-Life Assurances on Male Lives, approximately out of every 11 cases, in 7 the age next birthday, and in 4 the age last birthday was the "Nearest" age. The tabular ages at date of assurance were then supplied upon the assumption that these cases followed the same proportionate distributions; *i.e.*, in 7 cases out of 11 the age next birthday, and in the other cases the age last birthday, was assumed to be the "Nearest age." In a similar manner were treated the cases in the other categories (English Section, Endowment Assurances, &c.), the appropriate distributions being furnished from Appendix C.

(2) In the case of the second variety, "Year of Birth only given," it was evident that the year of birth, as recorded, was in a large proportion of cases an assumed date deduced by the contributing Offices (probably for valuation class-list purpose) by deducting the office age at entry from the calendar year of entry. As the office age at entry was not given in these cases, the only practicable course appeared to be to enter, as the age at entry for tabular purposes, the differences between the given year of birth and the calendar year of entry. A careful examination was made to prevent inconsistent dealings with different policies on the same life, and this examination confirmed the conclusion previously arrived at, it being found that, in a considerable number of cases, cards in respect of the same life were entered up with years of birth differing by unity. The total numbers of cards in this class were respectively: Male lives 15,140; Female lives 2,379.

(3) In the third variety, "Year of Birth and Office Age at Entry both given," it was again evident that the year of birth as recorded was an assumed one derived from the office age at entry and the calendar year of entry. The year of birth as recorded was therefore ignored, and the cases were treated precisely as if belonging to the first variety, "Office Age at Entry only given." The number of these cards was much smaller, viz., Male Lives 2,350; Female lives 290.

(4) In the fourth variety, "Date of Baptism only given," the date of birth was assumed to have been one month prior to that of baptism, and the nearest age at entry was deduced accordingly, and stamped upon the cards. The numbers were very small, viz., Male lives 220; Female lives 47.

Appendix F.

METHODS ADOPTED FOR DETERMINING AND RECORDING THE DURATIONS UPON THE CARDS.

First Stage.—Sorting according to mode of exit. The cards having been so sorted, those with no recorded mode of exit were then examined to see that they had no *date* of exit, in which case they were "Existing in 1893." If a date of exit was given they were set aside for inquiry of the Office as to mode of exit.

DEATHS AND WITHDRAWALS.

Second Stage.—Sorting according to date of exit as follows:—

(*a*) Month of exit later than month of entry.

(*b*), (*c*), (*d*) Month of exit same as month of entry.

(*e*) Month of exit earlier than month of entry.

(*f*) Neither day nor month of exit recorded.

The second group was then subdivided according to order of *day* of entry and exit, in like manner into (*b*) day of exit later, (*c*) day of exit same, and (*d*) day of exit earlier. The group (*f*) was also examined to see whether a year of exit was given. Cases where no date of exit whatever was given were then referred to the Offices for insertion of such date. Cases where the year of exit only was given were treated as "defective data." In the case of Deaths, groups (*a*), (*b*), and (*c*) were amalgamated, forming a group in which the difference between the calendar years of entry and exit gave the curtate duration at exit, and groups (*d*) and (*e*) were amalgamated, forming a group in which the difference between the calendar years of entry and exit was one greater than the curtate duration at exit. In the case of Withdrawals, however, the five groups (*a*), (*b*), (*c*), (*d*), and (*e*) were still kept separate, for reasons which will presently appear.

Third Stage.—Sorting according to year of entry. At this stage the "Old" became separated from the "New" Assurance cards ("Old" up to 1862 inclusive, "New" thereafter), and the packets were checked to see that the cards were of the right form. Any cards found to be of the wrong form were replaced by fresh cards, and cases of year of entry later than 1892 were excluded. The Existing were then ready for recording the durations at exit, and also all the "Old" cards for recording the durations in 1863.

Fourth Stage.—Sorting Deaths and Withdrawals into years of exit, under each of the different sub-sections so far formed. In the course of this process the following cards, representing cases emerging before the commencement of the experience, were excluded:—In groups (*a*), (*b*), or (*c*) (*Second Stage*), those with year of exit earlier than 1863, and in groups (*d*) or (*e*), those with year of exit 1863 or earlier. In the former groups also the mode of exit, on cards with year of exit 1893 or later, and in the latter groups

with year of exit later than 1893, was altered to "Existing," and the dates of exit cancelled.

Fifth Stage.—The groups of years of entry and years of exit were then amalgamated, according to the difference between such years. After this amalgamation had been carefully checked, the stamping of curtate duration at exit was effected.

WITHDRAWALS.

Sixth Stage.—It being necessary merely to obtain the cases emerging in the periods 0–2 months, 2–6 months, 6–8 months, and 8–12 months in the year of duration current at exit, the groups (*b*), (*c*), and (*d*) (*Second Stage*), which formed more than one-third of the whole body, were at once able to be stamped, groups (*b*) and (*c*) as belonging to period 0–2, and group (*d*) as belonging to period 8–12. The groups (*a*) and (*e*), being still kept separate, were sorted into months of entry; and

Seventh Stage.—The cards for each month of entry were further sorted according to month of exit.

Eighth Stage.—Within each of the groups (*a*) and (*e*) the packets of cards were then amalgamated according to differences of months. The cases where the difference in months was 2 months, 6 months, 8 months, were then split up according to *day*, and the resultant packets formed for stamping fractional durations, viz.:—

0–2	months,	stamped	(1).
2–6	,,	,,	(4).
6–8	,,	,,	(7).
8–12	,,	,,	(10).

Cases of exact differences 0 and 2 months were included in the group stamped (1); cases of exact differences 6 and 8 months were included in the group stamped (7).

[Experimental Data.]

WHOLE-LIFE ASSURANCES. PARTICIPATING AND NEW ASSURANCES

Distributions of Withdrawals, and Fractional Incidence

Curtate Duration	Monthly Incidence of Withdrawals in Year of Exit													
	0—½	0	1	½—1½	1½—2	0—2	2—2½	2½—3½	3	4	3½—4½	4½—5½	5½—6	2—6
0	...	...	...	...	2	*2*	1	9	34	16	3	2	5	*70*
1	10	403	373	21	5	*812*	2	10	8	9	12	1	2	*44*
2	11	258	182	37	26	*514*	12	18	7	1	3	5	7	*53*
3	17	155	94	44	23	*333*	11	14	3	4	5	6	5	*48*
4	16	90	76	28	13	*223*	9	8	1	2	8	7	3	*38*
0—4	**54**	**906**	**725**	**130**	**69**	**1,884**	**35**	**59**	**53**	**32**	**31**	**21**	**22**	**253**
5	16	77	41	38	21	*193*	7	12	4	1	7	3	2	*36*
6	12	57	33	22	8	*132*	4	11	2	1	7	4	1	*30*
7	9	38	21	19	3	*90*	4	12	5	1	2	1	2	*27*
8	9	37	34	20	7	*107*	4	6	1	2	6	6	6	*31*
9	10	34	17	10	5	*76*	3	9	1	2	4	2	2	*23*
5—9	**56**	**243**	**146**	**109**	**44**	**598**	**22**	**50**	**13**	**7**	**26**	**16**	**13**	**147**
10	6	28	25	13	3	*75*	3	2	...	...	3	6	1	*15*
11	5	24	17	10	4	*60*	2	5	2	...	2	1	2	*14*
12	3	23	14	9	7	*56*	3	1	...	1	...	2	3	*10*
13	3	16	9	7	3	*38*	1	3	1	...	5	2	...	*12*
14	3	14	10	7	1	*35*	1	1	...	...	4	4	2	*12*
10—14	**20**	**105**	**75**	**46**	**18**	**264**	**10**	**12**	**3**	**1**	**14**	**15**	**8**	**63**
15	2	7	11	8	1	*29*	2	3	...	...	1	2	...	*8*
16	3	15	2	2	2	*24*	1	1	1	...	...	...	1	*4*
17	1	9	3	5	...	*18*	2	3	...	...	...	...	...	*5*
18	1	8	4	3	...	*16*	...	1	...	...	...	...	2	*3*
19	2	2	4	2	1	*11*	1	...	1	...	2	1	...	*5*
15—19	**9**	**41**	**24**	**20**	**4**	**98**	**6**	**8**	**2**	**...**	**3**	**3**	**3**	**25**
20	1	4	3	...	...	*8*	...	...	1	...	...	2	...	*3*
21	...	2	...	...	1	*3*	...	...	...	...	1	...	...	*1*
22	...	4	...	1	...	*5*	...	1	...	...	...	1	...	*2*
23	...	3	1	1	...	*5*	...	...	...	...	...	...	...	...
24	...	...	1	...	...	*1*	...	...	...	...	...	1	...	*1*
20—24	**1**	**13**	**5**	**2**	**1**	**22**	**...**	**1**	**1**	**...**	**1**	**4**	**...**	**7**
25	...	...	...	...	...	...	...	...	...	...	...	1	...	*1*
26	...	...	...	...	...	...	...	...	...	...	...	...	...	...
27	...	...	...	...	...	...	...	...	...	...	...	...	...	...
28	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29	...	...	...	...	...	...	...	...	...	...	...	...	...	...
25—29	...	...	...	...	...	...	...	...	...	...	...	1	...	1
Totals	140	1,308	975	307	136	*2,866*	73	130	72	40	75	60	46	*496*

MALE LIVES BORN IN 1846. Appendix G.

NON-PARTICIPATING

EFFECTED 1863-1892.

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according to Curtate Duration
in Year of Exit

Monthly Incidence of Withdrawals in Year of Exit															Curtate Duration
6—6½	6	7	6½—7½	7½—8	6—8	8—8½	8½—9½	9	10	9½—10½	10½—11½	11½—12	8—12	Totals	
4	130	100	4	1	*239*	1	10	16	4	8	8	10	*57*	368	0
2	56	47	13	6	*124*	3	8	8	3	7	8	5	*42*	1,022	1
4	25	30	16	6	*81*	2	9	7	5	9	16	15	*63*	711	2
3	17	17	11	6	*54*	7	13	1	5	8	10	7	*51*	486	3
3	18	11	8	7	*47*	6	6	2	7	6	5	14	*46*	354	4
16	**246**	**205**	**52**	**26**	**545**	**19**	**46**	**34**	**24**	**38**	**47**	**51**	**259**	**2,941**	**0—4**
2	11	7	11	4	*35*	4	9	1	...	9	8	8	*39*	303	5
1	7	4	3	5	*20*	5	5	1	2	5	9	8	*35*	217	6
5	10	5	9	3	*32*	...	7	3	2	5	13	6	*36*	185	7
4	7	5	4	3	*23*	4	6	3	1	9	7	3	*33*	194	8
2	5	4	5	1	*17*	2	3	2	...	3	5	4	*19*	135	9
14	**40**	**25**	**32**	**16**	**127**	**15**	**30**	**10**	**5**	**31**	**42**	**29**	**162**	**1,034**	**5—9**
...	2	4	7	4	*17*	1	5	2	3	...	2	4	*17*	124	10
3	2	4	2	2	*13*	2	1	1	...	3	6	4	*17*	104	11
1	5	1	4	5	*16*	2	5	1	2	5	6	2	*23*	105	12
3	3	...	3	2	*11*	2	...	...	1	2	9	6	*20*	81	13
2	2	...	4	1	*9*	3	3	...	...	1	1	2	*10*	66	14
9	**14**	**9**	**20**	**14**	**66**	**10**	**14**	**4**	**6**	**11**	**24**	**18**	**87**	**480**	**10—14**
...	...	2	1	1	*4*	1	4	1	...	2	...	3	*11*	52	15
...	1	2	1	3	*7*	1	...	1	...	1	1	...	*4*	39	16
1	2	...	2	1	*6*	1	1	...	1	...	1	1	*5*	34	17
...	...	1	...	...	*1*	3	...	1	...	3	1	...	*8*	28	18
...	1	...	2	1	*4*	...	2	...	...	...	3	1	*6*	26	19
1	**4**	**5**	**6**	**6**	**22**	**6**	**7**	**3**	**1**	**6**	**6**	**5**	**34**	**179**	**15—19**
...	1	...	...	1	*2*	...	1	...	...	3	3	2	*9*	22	20
...	...	1	2	1	*4*	...	...	...	...	...	1	...	*1*	9	21
1	2	...	...	...	*3*	1	...	...	...	...	...	1	*2*	12	22
...	...	...	...	...	...	...	...	...	...	...	...	...	...	5	23
...	...	...	...	...	...	...	...	...	...	...	1	...	*1*	3	24
1	**3**	**1**	**2**	**2**	**9**	**1**	**1**	...	...	**3**	**5**	**3**	**13**	**51**	**20—24**
...	1	...	...	1	*2*	...	...	...	...	...	...	...	...	3	25
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	26
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	27
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	28
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	29
...	1	...	...	1	2	...	...	...	...	...	...	...	...	3	25—29
41	308	245	112	65	*771*	51	98	51	36	89	124	106	*555*	4,688	Totals

Distribution of Withdrawals, according to Curtate Duration and Fractional Incidence in Year of Exit.

WHOLE-LIFE PARTICIPATING "NEW" ASSURANCES.

[Experimental Data.] MALE LIVES BORN IN 1862. Appendix H.

Curtate Duration	Monthly Incidence of Withdrawals in Year of Exit.																											Curtate Duration
	0—½	0	1	½—1½	1½—2	0—2	2—2½	2½—3½	3	4	3½—4½	4½—5½	5½—6	2—6	6—6½	6	7	6½—7½	7½—8	6—8	8—8½	8½—9½	9	10	9½—10½	10½—11½	11½—12	8—12	Totals	
0	..	..	..	1	..	1	..	..	9	5	3	4	2	23	8	72	52	2	..	134	1	4	10	..	3	5	5	28	186	0
1	7	243	165	8	1	424	2	4	4	..	6	1	..	17	2	33	27	6	3	71	..	1	2	1	1	7	2	14	526	1
2	3	89	66	13	9	180	4	3	2	..	4	1	1	15	3	11	7	3	2	26	2	1	2	..	3	6	4	18	239	2
3	11	43	14	11	4	83	4	4	1	1	5	3	..	18	..	6	4	5	4	19	1	6	2	1	1	6	6	23	143	3
4	..	17	21	4	7	49	2	3	..	3	2	4	1	15	1	2	1	2	..	6	..	4	2	1	2	2	4	15	85	4
0—4	21	392	266	37	21	737	12	14	16	9	20	13	4	88	14	124	91	18	9	256	4	16	18	3	10	26	21	98	1,179	0—4
5	6	19	11	5	4	45	4	3	..	1	1	1	..	10	7	1	1	1	1	11	2	2	1	..	2	2	4	13	79	5
6	7	6	8	3	1	25	2	1	..	..	3	1	..	7	2	..	..	1	1	4	..	1	..	..	4	2	3	10	46	6
7	1	3	3	6	1	14	..	..	..	..	2	..	1	3	3	..	..	..	..	3	..	1	..	..	1	2	1	5	25	7
8	1	6	2	2	1	12	..	..	..	..	2	1	..	3	..	1	1	..	..	2	..	..	..	..	2	..	1	3	20	8
9	2	2	..	1	1	6	..	1	..	..	1	..	..	2	1	..	..	..	1	2	1	..	..	..	..	1	1	3	13	9
5—9	17	36	24	17	8	102	6	5	..	1	9	3	1	25	13	2	2	2	3	22	3	4	1	..	9	7	10	34	183	5—9
0—9	38	428	290	54	29	839	18	19	16	10	29	16	5	113	27	126	93	20	12	278	7	20	19	3	19	33	31	132	1,362	0—9

MALE LIVES BORN IN 1816.

0	..	..	..	..	..	..	..	..	4	..	2	..	1	7	..	9	4	1	..	14	..	..	1	1	..	1	..	3	24	0
1	3	26	35	5	..	69	..	2	1	..	2	1	..	6	4	5	5	2	..	16	..	1	..	..	..	1	1	3	94	1
2	2	17	19	1	1	40	1	..	..	..	1	2	..	4	..	1	3	1	2	7	1	..	..	..	..	1	1	3	54	2
3	1	6	6	5	2	20	..	1	2	..	1	..	1	5	1	1	1	..	..	3	..	..	..	..	..	..	..	..	28	3
4	..	5	1	5	..	11	..	..	..	..	1	2	1	4	..	..	..	1	..	1	..	..	..	..	..	3	1	4	20	4
0—4	6	54	61	16	3	140	1	3	7	..	7	5	3	26	5	16	13	5	3	41	1	1	1	1	..	6	3	13	220	0—4
5	2	2	4	..	..	8	1	3	..	..	1	2	1	8	..	2	1	1	1	5	..	1	..	..	..	2	1	4	25	5
6	..	5	5	1	1	12	1	1	..	1	1	..	..	4	..	..	1	1	..	2	..	..	..	..	1	3	..	4	22	6
7	3	3	1	3	1	11	..	..	..	..	..	..	..	..	1	1	2	..	..	4	2	..	..	..	..	..	..	2	17	7
8	2	3	1	1	..	7	1	1	..	..	2	..	..	4	..	..	..	1	..	1	..	2	..	..	..	..	1	3	15	8
9	..	2	1	2	..	5	..	1	..	..	..	1	..	2	..	..	..	2	..	2	..	..	..	..	..	1	1	2	11	9
5—9	7	15	12	7	2	43	3	6	..	1	4	3	1	18	1	3	4	5	1	14	2	3	..	..	1	6	3	15	90	5—9
0—9	13	69	73	23	5	183	4	9	7	1	11	8	4	44	6	19	17	10	3	55	3	4	1	1	1	12	6	28	310	0—9

FEMALE LIVES BORN IN 1850-1865.

0	..	..	2	..	..	2	..	3	9	10	3	2	11	38	1	48	69	3	..	121	..	3	6	7	1	6	23	46	207	0
1	14	241	278	11	3	547	1	5	4	7	3	5	3	28	1	29	27	5	5	67	1	3	1	4	4	5	18	36	678	1
2	12	102	133	20	6	273	2	6	1	1	3	4	2	19	2	8	14	5	6	35	..	2	1	1	4	15	17	40	367	2
3	10	40	55	23	5	133	3	5	..	..	3	1	3	15	..	5	9	7	2	23	2	2	2	3	5	7	8	29	200	3
4	5	24	29	13	6	77	5	2	1	..	3	5	1	17	2	3	3	4	1	13	2	1	..	1	5	1	13	23	130	4
0—4	41	407	497	67	20	1,032	11	21	15	18	15	17	20	117	6	93	122	24	14	259	5	11	10	16	19	34	79	174	1,582	0—4
5	6	22	24	8	9	69	2	5	1	..	2	3	4	17	..	2	3	5	1	11	1	3	..	..	1	5	6	8	24	121	5
6	4	14	10	8	2	38	2	4	1	1	1	2	3	14	..	2	..	2	1	5	2	1	..	..	2	4	6	15	72	6
7	8	13	6	4	4	35	..	..	..	..	4	4	1	9	2	2	3	3	1	11	..	4	..	..	2	4	8	18	73	7
8	2	7	7	3	1	20	..	2	..	..	4	1	1	8	2	..	1	2	1	6	..	1	..	..	2	8	7	18	52	8
9	1	2	4	6	2	15	1	1	..	..	1	2	2	7	..	..	..	2	2	4	..	2	..	..	1	1	2	6	32	9
5—9	21	58	51	29	18	177	5	12	2	1	12	12	11	55	4	6	7	14	6	37	3	11	..	1	12	23	31	81	350	5—9
0—9	62	465	548	96	38	1,209	16	33	17	19	27	29	31	172	10	99	129	38	20	296	8	22	10	17	31	57	110	255	1,932	0—9

WHOLE-LIFE PARTICIPATING "NEW" ASSURANCES.

Comparative Statement of Fractional Duration of Withdrawals, as estimated by the Exact Duration Method, the Nearest Duration Method, and a Modified Nearest Duration Method; also of the Number of Cases entering upon each Year of Assurance.

MALE LIVES—BORN IN 1862.

Curtate Duration	Number of Cases entering on Year of Assurance	Exact Duration Method *		Nearest Duration Method †			Modified Nearest Duration Method			Curtate Duration
		Duration		Duration	Deviation		Duration	Deviation		
		Years	Months	Years	Years	Months	Years	Years	Months	
0	5,999	110	0	162	+52	0	110	...	...	0
1	5,163	93	11	85	— 8	11	94	+ 0	1	1
2	4,109	50	8	44	— 6	8	51	+ 0	4	2
3	3,373	43	0	42	— 1	0	45	+ 2	0	3
4	2,729	26	0	21	— 5	0	25	— 1	0	4
0—4	21,373	323	7	354	+30	5	325	+ 1	5	0—4
5	2,203	23	2	24	+ 0	10	25	+ 1	10	5
6	1,663	14	9	14	— 0	9	16	+ 1	3	6
7	1,235	8	3	8	— 0	3	9	+ 0	9	7
8	881	5	11	5	— 0	11	5	— 0	11	8
9	590	4	9	5	+ 0	3	5	+ 0	3	9
5—9	6,572	56	10	56	— 0	10	60	+ 3	2	5—9
0—9	27,945	380	5	410	+29	7	385	+ 4	7	0—9

MALE LIVES—BORN IN 1816.

0	1,828	13	3	17	+ 3	9	12	— 1	3	0
1	1,791	19	1	19	— 0	1	19	— 0	1	1
2	1,676	11	5	10	— 1	5	11	— 0	5	2
3	1,594	5	2	3	— 2	2	4	— 1	2	3
4	1,546	6	11	5	— 1	11	6	— 0	11	4
0—4	8,435	55	10	54	— 1	10	52	— 3	10	0—4
5	1,489	9	8	9	— 0	8	9	— 0	8	5
6	1,436	6	11	6	— 0	11	7	+ 0	1	6
7	1,383	4	4	6	+ 1	8	5	+ 0	8	7
8	1,343	4	7	4	— 0	7	5	+ 0	5	8
9	1,283	4	2	4	— 0	2	4	— 0	2	9
5—9	6,934	29	8	29	— 0	8	30	+ 0	4	5—9
0—9	15,369	85	6	83	— 2	6	82	— 3	6	0—9

FEMALE LIVES—BORN 1850-1865

0	6,037	127	6	167	+39	6	124	— 3	6	0
1	5,802	126	9	103	—23	9	126	— 0	9	1
2	4,680	86	5	75	—11	5	88	+ 1	7	2
3	3,891	54	6	52	— 2	6	57	+ 2	6	3
4	3,366	40	3	36	— 4	3	41	+ 0	9	4
0—4	23,776	435	5	433	— 2	5	436	+ 0	7	0—4
5	2,925	39	9	35	— 4	9	40	+ 0	3	5
6	2,501	24	1	20	— 4	1	24	— 0	1	6
7	2,136	28	9	29	+ 0	3	30	+ 1	3	7
8	1,831	24	6	24	— 0	6	25	+ 0	6	8
9	1,558	11	8	10	— 1	8	10	— 1	8	9
5—9	10,951	128	9	118	—10	9	129	+ 0	3	5—9
0—9	34,727	564	2	551	—13	2	565	+ 0	10	0—9

* Cases of Withdrawal at the precise points 0, 3, 6, 9 (as recorded) being treated throughout as of durations 1, 4, 7, 10 respectively.

† Cases recorded as Withdrawals at the precise point 6 being treated as of duration 7, and classed with those falling in the second half of the year.

ENDOWMENT ASSURANCES PARTICIPATING AND NON-PARTICIPATING.

NEW ASSURANCES—EFFECTED 1863-1892.

[Experimental Data.] Appendix K.

Distribution of Withdrawals, according to Curtate Duration, and Fractional Incidence in Year of Exit

Curtate Duration	Monthly Incidence of Withdrawals in Year of Exit.																													Curtate Duration
	0—½	0	1	½—1½	1½—2	0—2	2—2½	2½—3½	3	4	3½—4½	4½—5½	5½—6	2—6	6—6½	6	7	6½—7½	7½—8	6—8	8—8½	8½—9½	9	10	9½—10½	10½—11½	11½—12	8—12	Totals	
MALE LIVES BORN IN 1846.																														
0	..	..	..	..	..	..	..	2	5	5	..	2	2	16	1	23	25	2	..	51	..	2	2	4	2	1	1	12	79	0
1	2	56	65	..	1	124	2	..	3	2	..	3	..	10	1	7	16	..	..	24	..	..	1	..	1	..	2	4	162	1
2	4	32	42	11	6	95	3	4	..	1	..	1	1	10	..	3	4	3	..	10	..	1	..	1	3	2	3	10	125	2
3	2	16	25	9	8	60	..	2	1	..	..	..	..	3	1	6	5	4	..	16	1	2	..	..	..	1	5	9	88	3
4	4	8	17	8	2	39	..	3	..	..	2	2	1	8	1	6	1	2	1	11	..	..	..	..	1	2	6	9	67	4
0—4	12	112	149	28	17	318	5	11	9	8	2	8	4	47	4	45	51	11	1	112	1	5	3	5	7	6	17	44	521	0—4
5	2	11	10	6	3	32	5	1	..	..	..	1	..	7	1	5	1	..	2	9	..	..	1	1	3	1	3	9	57	5
6	..	7	6	3	..	16	..	1	..	..	..	1	2	4	..	1	..	2	1	4	2	1	..	..	2	2	1	8	32	6
7	2	7	1	1	1	12	..	1	..	..	2	2	..	5	1	1	..	..	2	4	1	..	1	1	1	1	1	6	27	7
8	3	4	1	4	1	13	..	1	..	..	1	..	..	2	..	3	..	..	1	4	..	..	..	..	1	2	1	4	23	8
9	..	4	5	..	..	9	1	..	1	..	1	1	..	4	..	..	1	..	1	2	..	1	..	1	1	..	..	3	18	9
5—9	7	33	23	14	5	82	6	4	1	..	4	5	2	22	2	10	2	2	7	23	3	2	2	3	8	6	6	30	157	5—9
0—9	19	145	172	42	22	400	11	15	10	8	6	13	6	69	6	55	53	13	8	135	4	7	5	8	15	12	23	74	678	0—9
MALE LIVES BORN IN 1862.																														
0	..	..	..	..	..	..	3	6	9	6	4	1	2	31	3	42	65	5	2	117	..	3	5	2	4	2	3	19	167	0
1	10	114	181	6	3	314	1	2	4	1	1	4	4	17	1	8	21	2	3	35	2	9	2	2	..	6	4	25	391	1
2	3	41	57	7	3	111	1	1	1	1	3	6	..	13	2	9	12	4	3	30	1	1	..	2	3	4	4	15	169	2
3	3	16	22	15	3	59	2	3	3	..	4	2	..	14	2	1	2	4	..	9	1	..	1	2	1	3	1	9	91	3
4	6	10	15	5	3	39	..	4	1	..	1	3	1	10	..	2	1	2	2	7	..	3	2	..	4	2	..	11	67	4
0—4	22	181	275	33	12	523	7	16	18	8	13	16	7	85	8	62	101	17	10	198	4	16	10	8	12	17	12	79	885	0—4
5	4	6	9	5	2	26	..	2	1	2	2	3	..	10	..	..	2	2	..	4	1	1	..	..	..	4	..	6	46	5
6	2	4	5	5	..	16	2	1	..	..	..	..	..	3	..	1	1	1	1	4	..	..	..	1	1	..	..	2	25	6
7	..	5	3	1	2	11	..	..	..	..	..	1	..	1	..	1	..	..	..	1	..	2	..	..	..	1	..	3	16	7
8	..	..	2	..	..	2	..	..	..	1	1	..	..	2	..	..	..	..	..	..	..	1	..	..	3	..	..	4	8	8
9	..	2	..	..	..	2	..	..	1	..	..	..	..	1	..	..	..	..	1	1	..	..	..	..	..	1	..	1	5	9
5—9	6	17	19	11	4	57	2	3	2	3	3	4	..	17	..	2	3	3	2	10	1	4	..	1	4	6	..	16	100	5—9
0—9	28	198	294	44	16	580	9	19	20	11	16	20	7	102	8	64	104	20	12	208	5	20	10	9	16	23	12	95	985	0—9

Appendix L.

ENDOWMENT ASSURANCES.

PARTICIPATING AND NON-PARTICIPATING.

NEW ASSURANCES—EFFECTED 1863-1892.

Comparative Statement of Fractional Duration of Withdrawals, as estimated by the Exact Duration Method, the Nearest Duration Method, and the Modified Nearest Duration Method, also of the Number of Cases entering upon each Year of Assurance

Curtate Duration	Number of Cases Entering on Year of Assurance	Exact Duration Method*		Nearest Duration Method†			Modified Nearest Duration Method			Curtate Duration
		Duration		Duration	Deviation		Duration	Deviation		
		Years	Months	Years	Years	Months	Years	Years	Months	
MALE LIVES BORN IN 1846										
0	2,581	45	5	63	+17	7	44	— 1	5	0
1	2,299	31	1	28	— 3	1	29	— 2	1	1
2	2,084	25	7	20	— 5	7	25	— 0	7	2
3	1,863	23	5	25	+ 1	7	24	+ 0	7	3
4	1,706	20	11	20	— 0	11	20	— 0	11	4
0—4	10,533	146	5	156	+ 9	7	142	— 4	5	0—4
5	1,543	17	6	18	+ 0	6	18	+ 0	6	5
6	1,399	11	11	12	+ 0	1	12	+ 0	1	6
7	1,266	10	2	10	— 0	2	10	— 0	2	7
8	1,136	7	7	8	+ 0	5	8	+ 0	5	8
9	1,033	5	8	5	— 0	8	5	— 0	8	9
5—9	6,377	52	10	53	+ 0	2	53	+ 0	2	5—9
0—9	16,910	199	3	209	+ 9	9	195	— 4	3	0—9
MALE LIVES BORN IN 1862										
0	5,278	94	2	136	+41	10	91	— 3	2	0
1	5,096	73	5	60	—13	5	75	+ 1	7	1
2	4,020	44	8	45	+ 0	4	44	— 0	8	2
3	3,184	22	0	18	— 4	0	21	— 1	0	3
4	2,524	19	9	18	— 1	9	20	+ 0	3	4
0—4	20,102	254	0	277	+23	0	251	— 3	0	0—4
5	1,969	12	10	10	— 2	10	11	— 1	10	5
6	1,491	5	10	6	+ 0	2	6	+ 0	2	6
7	1,027	4	6	4	— 0	6	5	+ 0	6	7
8	711	4	1	4	— 0	1	4	— 0	1	8
9	457	2	1	2	— 0	1	2	— 0	1	9
5—9	5,655	29	4	26	— 3	4	28	— 1	4	5—9
0—9	25,757	283	4	303	+19	8	279	— 4	4	0—9

* Cases of Withdrawal at the precise points 0, 3, 6, 9 (as recorded) being treated throughout as of durations 1, 4, 7, 10 respectively.

† Cases at the precise point 6 being treated as of duration 7, and classed with those falling in the second half of the year.

Appendix M.

AS TO THE *RATIONALE* OF THE MODIFIED NEAREST DURATION METHOD,

WITH AN INVESTIGATION OF THE AMOUNT OF THE TABULAR ERROR.

The general principles upon which the method was based may be enunciated as follows:—If r be the days of grace (stated as the fraction of a year), then the duration of the lapses, and consequently of the greater portion of the withdrawals, arising in a given policy-year, will be shifted from the durations 0, ·25, ·50 and ·75, to the later durations r, $(\cdot25+r)$, $(\cdot50+r)$, and $(\cdot75+r)$. The policy-year may now be considered as divided into two periods; the first of which extends from 0 to $2r$, the central point of which, r, represents the true fractional duration of the yearly lapses (inclusive of the days of grace), and the average fractional duration of the surrenders included in the period. The second period extends from $2r$ to 1, the central point of which, $(\cdot5+r)$, represents the true duration of the half-yearly lapses; is equidistant from the true durations of the quarterly lapses arising at the points $(\cdot25+r)$ and $(\cdot75+r)$; and also represents the average duration of the surrenders included in the period, so far as they are equally distributed over its duration.

The cases of withdrawal arising in the first period may thus be conveniently and accurately tabulated as of fractional duration r; whilst those arising in the second period may be dealt with, according to the general principles of the Nearest Duration Method, by referring the cases recorded between durations $2r$ and $(\cdot5+r)$ to the beginning of the period, and treating them all as of duration $2r$; by refefring the cases between durations $(\cdot5+r)$ and 1 to the end of the period, and treating them all as of duration 1; and finally, by referring cases arising at the precise duration $(\cdot5+r)$ alternately to the beginning and end of the period.

It is, however, to be observed, that it will be equally consistent with the principles of the Nearest Duration Method, if the assumed durations, at the commencement and end of the second period, differ from those above stated, so long as the essential condition is observed, that the *sum* of such assumed durations is equal to $(1+2r)$, with an average or central duration of $(\cdot5+r)$. This necessary condition will obtain, for instance, if the assumed durations are respectively taken as r and $(1+r)$; and these will be the most convenient assumptions in practice, for the commencing duration r, as assumed for the

second period, will then coincide with the average duration adopted for the whole of the cases included in the first period.

These assumptions will be carried into effect by referring the whole of the cases arising in both periods to the beginning and end of the year, according as they withdrew prior, or subsequently to the assumed central point of duration ($\cdot5+r$), cases arising precisely at the latter duration being distributed equally to the beginning and end of the year. A further period of duration, equivalent to r, must then be added to the whole of the cases; and this can most conveniently be effected by transferring, from the cases already referred to the beginning of the year, a number which represents the proportion r of the whole number of withdrawals in the year, and adding such cases to those already referred to the end of the year. In order to avoid the introduction of fractional exposures, the proportion so transferred must be taken to the nearest integer (*see* note to Table III., p. 45).

In the practical application of these principles to the case of the present experience, it will be observed that, as the lapses are in some cases recorded with exclusion, and in others with inclusion, of the days of grace, the half-yearly lapses, as recorded upon the data cards, would arise at the two points of duration, ·5 and ($\cdot5+r$), respectively. It was therefore necessary, in order to ensure a uniform treatment of these varying records, slightly to modify the method above indicated, by comprising all durations between ·5 and ($\cdot5+2r$), both inclusive, in a *central group*. Withdrawals arising prior to duration ·5 were then referred to the beginning of the year; those arising after duration ($\cdot5+2r$) were referred to the end of the year; those included in the central group were referred alternately to the beginning and end of the year; and a further exposure of r was added to the whole of the cases.

As the days of grace extended, in the great majority of the cases included in the experience, over one month, that value was adopted for r; and each policy-year would thus be divided into three groups, as under:—

Group (i) 0–6 Months—cases referred to the beginning of the year;
„ (ii) 6–8 „ } cases referred alternately to the beginning and end of the year;
„ (iii) 8–12 „ cases referred to the end of the year;—

the additional duration of one month being obtained by further transferring one-twelfth, disregarding fractions, of the total number of cases included in all three groups, from the beginning to the end of the year.

It will be seen, from the above investigation, that the addition of one month's exposure, in arriving at the tabular duration of all the cases falling in the year, does not exclusively represent the days of grace in lapsed cases. By the principles upon which the method is based, the considerable number of withdrawals, arising at the precise durations of one month and seven months (mainly in respect of yearly and half-yearly lapses) have their durations correctly tabulated; whilst the other withdrawals (whether by lapse or surrender) are so tabulated as to introduce compensating errors, so far as the cases, arising at points equidistant (on different sides) from duration one month, and duration seven months, are respectively equal in number.

The investigation of the error involved in the application of the method may thus be theoretically stated :—

Representing by w_t the number of withdrawals in any policy-year, having exactly t months' duration (inclusive of the days of grace), where t may have any value, fractional or integral, between 0 and 12, we have, for the aggregate true exposures of all the withdrawals falling in the year, the general expression

$$\sum_{t=0}^{t=12} (w_t \times t).$$

Considering now the cases falling in the first group (0—2 months' duration), we have, for the aggregate true exposure of the cases in the group,

$$\sum_{t=0}^{t=2} (w_t \times t).$$

The *tabular* duration, which gives to all cases in the group a uniform duration of one month, is equal to

$$\sum_{t=0}^{t=2} (w_t),$$

and the aggregate error in the exposures of the group is thus

$$= \sum_{t=0}^{t=2} [w_t(1-t)].$$

It is evident that the large number of cases arising (by yearly lapse) at the central duration of one month, are correctly tabulated; and that, so far as the cases arising at equidistant points on opposite sides of this central duration are equal in number, the sum of the errors will be zero.

Considering now the central group (6—8 months' duration), we have, similarly, for the true aggregate exposures,

$$\sum_{t=6}^{t=8} (w_t \times t).$$

By the method of tabulation followed, these cases are referred in equal parts to the beginning and end of the year, and one months' exposure is then added to the whole of the cases in the group; the tabular duration is thus equal to a mean duration in each case of $\left(\frac{0+12}{2}+1\right)$, or 7 months. The aggregate tabular duration of cases included in the group is thus equal to

$$\sum_{t=6}^{t=8} (7w_t)$$

and the aggregate error in the group is thus

$$= \sum_{t=6}^{t=8} [(7-t)(w_t)].$$

Here, again, it is evident that the considerable number of cases arising (by half-yearly lapse) at the central duration of 7 months are correctly tabulated; and that, so far as the cases arising at equidistant points on opposite sides of this central duration are equal in number, the sum of the errors will be zero.

[It may be added that, so far as the number of withdrawals in the interval 6–7 months differs from that in the interval 7–8 months, the adoption of a central *group*,—including all durations from 6 to 8 months, half the cases falling in which are given a tabular duration of 1 month, and the other half of 13 months,—evidently gives a closer approximation to the true duration of the cases included in the group, than the alternative plan of taking a central *point* of 7 months' duration, at which precise duration cases are referred alternately to durations of 1 month and 13 months, whilst all durations below 7, are classed as 1 month, and all above 7, as 13 months.]

The second group (2–6 months' duration) and the fourth group (6–12 months' duration) may be considered together; and the durations may most conveniently be reckoned from the central point of 7 months. We then have, for the aggregate true exposures:—

Second Group:— $\sum_{t=5}^{t=1}[(w_{7-t})(7-t)]$ *Fourth Group:—* $\sum_{t=1}^{t=5}[(w_{7+t})(7+t)]$

The tabular durations are, in the second group, (0+1) months, and, in the fourth group, (12+1) months, and amount in the aggregate to

$$\sum_{t=5}^{t=1}(w_{7-t}) \quad \text{and} \quad \sum_{t=1}^{t=5}(13w_{7+t}).$$

The aggregate errors in the two groups thus amount to

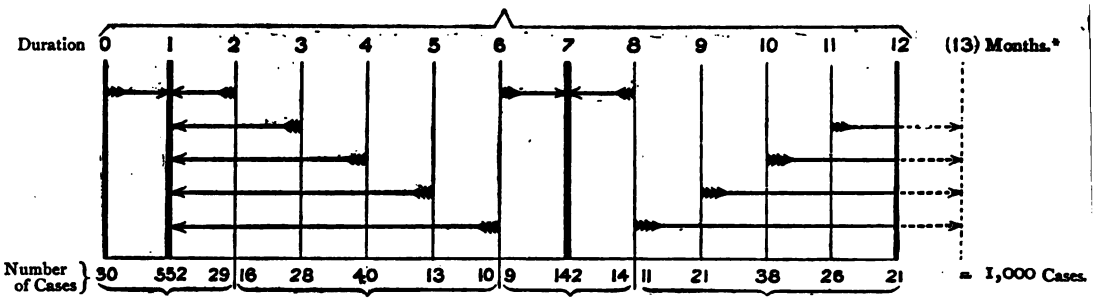
$$\sum_{t=5}^{t=1}[(w_{7-t})(t-6)] \quad \text{and} \quad \sum_{t=1}^{t=5}[(w_{7+t})(6-t)].$$

It is evident that, taking two points w_{7-t} and w_{7+t}, equidistant from the central point of duration, the respective errors $(t-6)$ and $(6-t)$ are equal in amount but contrary in sign, so that, if w_{7-t} and w_{7+t} are equal numbers, the sum of the errors will be zero.

[An alternative method of tabulating the withdrawals falling in the second, third, and fourth groups (2–12 months' duration), would have been, to give all the cases included in the group an average duration of 7 months. As regards the central group (6–8 months) this method would have given results identical with that adopted. As regards the second group (2–6 months), and the fourth group (8–12 months), the alternative method would, however, have introduced the *maximum* error at the end of the year, where (from the preponderance of surrenders) the number of cases would also tend to a maximum; whilst, by the method adopted, the *minimum* error is throughout introduced, where the number of cases is at a maximum.]

In the following graphic illustration of the application of the method, the distribution of 1000 cases of withdrawal over a typical policy-year is shown, according to the nearest month of actual duration, the figures being deduced from the distribution of the 4,688 cases shown in the total line of Appendix G.

—— MODIFIED NEAREST DURATION METHOD. ——



Grouped Numbers	611 × 1		107 × 1		165 × 7		117 × 13	
Tabular Duration	611	+	107	+	1,155	+	1,521	= 3,394 Months
True Duration	610¼	+	402½	+	1,158⅜	+	1,192½	= 3,364 ,,
Total Error	+ ¾	−	295½	−	3⅜	+	328½	= +30 ,,
Average Error	+ 1/815	−	2⅜	−	1/44	+	2⅛	= +3/1000 ,,
	+(0 days)	−	(82½ days)	−	(⅓ day)	+	(84 days)	= +(9/10 day)

* It will be understood that the attribution of 13 months' tabular duration to the cases having actual durations of upwards of 8 months, does not involve the prolongation of their exposures into the following policy-year, all durations being included strictly in the year in which they arise; so that, for instance, 12 cases of withdrawal, each with a tabular duration of 13 months, would be treated as equivalent to 13 cases, exposed throughout the policy-year.

The arrows show graphically the reference of the cases to the points of duration 1, 7 and 13; the length of the respective arrows show the extent of the error involved in the tabular duration; whilst equal compensatory errors are shown by arrows pointing to the right and left hand on the same horizontal line. Below the diagram are shown the grouped members in respect of durations (0—2), (2—6), (6—8), and (8—12) months respectively; and the tabular and true durations are then deduced and compared, with the resulting error in each group.

The tabular duration would in practice be deduced, by one or other of the following processes, to the nearest integer :—

(i) by arithmetical computation :—

$$\frac{165}{2}+117+\frac{1000}{12}=83+117+83=283 \text{ years (3,396 months).}$$

(ii) by transfer (*a*) to the beginning of the year, (duration 0) of $611+107+\frac{165}{2}=800$ cases; (*b*) to the end of the year, of $117+\frac{165}{2}=200$ cases; and (*c*) by further transfer, from the beginning to the end of the year, of $\frac{1000}{12}=83$ cases. The resulting tabular duration is thus $(200+83)=283$ years, as before.

It will be seen that, as might be anticipated by the above theoretical investigation, the errors in the group (0—2) months and (6—8) months are quite insignificant; whilst the aggregate errors in the other groups (2—6) months and (8—12) months, $-295\frac{1}{2}$ months and $+328\frac{1}{2}$ respectively, are contrary in sign, but do not materially differ in amount, the sum of the errors in these two groups being +33 months only, in respect of 224 cases. It will be further seen that this error arises solely from the number of cases in the group (8—12) months, 117, being greater than the number in the group (2—6) months, 107.

This excess in the group (8—12) months will usually be found in the third and following years of assurance (*see* Appendix G, pp. 92, 93), and no doubt chiefly arises from the tendency of the cases of surrender towards the end of the policy-year; and it is to be remarked that the amount of the error thus involved is reduced, in proportion as the cases of surrender congregate towards the end of the policy-year, where the difference between the tabular and true duration is at its minimum value.

Appendix N.

ENDOWMENT ASSURANCE EXPERIENCE.

COMBINED "OLD" AND "NEW" ASSURANCES.—WITH AND WITHOUT PROFITS.

SELECT TABLES. ALL AGES AT ENTRY.

Distribution over last Policy Year of Assurance of 10,000 Cases maturing on the Quinquennial Birthdays

50, 55, 60, 65.

Interval by which Maturity follows Policy Anniversary.	Cases Maturing on Actual Birthday $5x$.				Interval by which Maturity follows Policy Anniversary.
	50.	55.	60.	65.	
Maturities falling in Tabular Year of Age ($5x-1$) to ($5x$).					
6- 7 Mos.	186	132	261	15	6- 7 Mos.
7- 8 Mos.	176	127	281	29	7- 8 Mos.
8- 9 Mos.	161	103	302	44	8- 9 Mos.
9-10 Mos.	156	110	213	15	9-10 Mos.
10-11 Mos.	186	78	232	39	10-11 Mos.
11-12 Mos.	188	147	217	39	11-12 Mos.
Total	1,053	697	1,506	181	All ages. 3,437
Mean Duration in Tabular Year of Exit	Mos. 9·017	Mos. 8·953	Mos. 8·849	Mos. 9·334	Mos. 8·947
Maturities falling in Tabular Year of Age ($5x$) to ($5x+1$).					
0- 1 Mos.	889	469	1,139	144	0- 1 Mos.
1- 2 Mos.	354	186	447	59	1- 2 Mos.
2- 3 Mos.	281	166	276	56	2- 3 Mos.
3- 4 Mos.	249	134	305	42	3- 4 Mos.
4- 5 Mos.	210	122	288	39	4- 5 Mos.
5- 6 Mos.	244	112	323	29	5- 6 Mos.
Total	2,227	1,189	2,778	369	All ages. 6,563
Mean Duration in Tabular Year of Exit	Mos. 2·172	Mos. 2·155	Mos. 2·185	Mos. 2·121	Mos. 2·172
Grand Total	3,280	1,886	4,284	550	10,000

Appendix O.

METHODS OF DEALING WITH CASES OF DEFECTIVE DATA AS TO DATE OF EXIT.

In some cases under the several modes of exit (Death, Withdrawal, or Termination) the records in the Companies' books only sufficed to supply the calendar year of exit, and in order to obtain an approximation to the records in a form adapted to the compilation of the experience, certain assumptions became requisite. These were as follows:—

In the Whole-Life Class (Male Lives), which formed the major portion of the Assurances, the Deaths and Withdrawals were respectively sorted into six groups, according to the month of entry, as under:

	Months of Entry.	Mean date of Entry.
(i)	January and February	1 February
(ii)	March and April	1 April
(iii)	May and June	1 June
(iv)	July and August	1 August
(v)	September and October	1 October
(vi)	November and December	1 December

Deaths.—In these cases it was only required to mark the curtate duration at exit upon the cards, and this was determined upon the basis of an assumed uniform distribution of deaths through the calendar year, so that in group

(i)	The calendar year of exit was modified by	— 1,	in 1 case out of 12;
(ii)	do.	do.	3 cases out of 12;
(iii)	do.	do.	5 do.
(iv)	do.	do.	7 do.
(v)	do.	do.	9 do.
(vi)	do.	do.	11 do.

Withdrawals.—Here it was found from the typical data (see Appendices G, H, and K) that about half the cases were lapses on the anniversary, and accordingly, in each of the groups, half the cases were treated as such and were marked with the curtate duration, calculated from the difference between the calendar years of entry and exit; the fractional duration being recorded as W(1).

For the remaining half of the cases, an assumption was made of an even distribution through the calendar year; and the cases were so divided that, when stamped with an integral duration and the fractional duration W(1), they would supply in the aggregate a correct mean duration for the whole of the cases of the section.

This was carried out by means of the modifications set out in the following Schedule:—

Tabular Distribution of an assumed body of 144 Withdrawals in a given Calendar Year over the Year of Assurance current at exit.

Entrants in	Mean Date of Entry	Modified Central Date of Policy-Year 7 Months later	Number of Cases in which the Year of Exit must be				Hence Modify Year of Exit	
			Not Modified.		Modified by −1	Modified by +1		
			Lapses on Anniversary	Other Withdrawals			By	In 1 Case out of
Jan., Feb.	1 Feb.	1 Sept.	12	8	....	4	+1	6
Mar., April	1 April	1 Nov.	12	10	...	2	+1	12
May, June	1 June	1 Jan.*	12	12	...	...	...	...
†July, Aug.	1 Aug.	1 Mar.*	12	10	2	...	−1	12
Sept., Oct.	1 Oct.	1 May*	12	8	4	...	−1	6
Nov., Dec.	1 Dec.	1 July*	12	6	6	...	−1	4

* In following calendar year.

† Example.—In dealing with a group of 24 cases, effected between 1st July and 31st August, 1864, and emerging by Withdrawal in the calendar year 1884, the central date of entry was first taken as 1st August, 1864. In accordance with the principles of the "Modified Nearest Duration Method," the *nearest* points of reference (for cases withdrawing in 1884) were then taken either as 1st September, 1884, or 1st September, 1883. It was then assumed (*a*) that 12 cases withdrew by yearly lapse, and thus passed out of observation on 1st September, 1884; (*b*) that 10 further cases withdrew between 1st March and 31st December, 1884, with "nearest" duration as at 1st September, 1884, and (*c*) that the remaining 2 cases withdrew between 1st January and 29th February, 1884, with "nearest" duration as at 1st September, 1883.

Note.—As far as practicable, the "cycles" for modification were applied within each group of cases, at the same age at date of assurance and duration.

A similar plan was followed in dealing with the cards relating to other classes of assurance, but the number of cases with defective date of exit being smaller, the cards were divided into groups embracing three months in lieu of two.

Terminations.—In dealing with these cases, regard was had to the class of assurance to which the particulars related, so that—

Temporary Assurance cases were treated as emerging on the policy anniversary falling in the calendar year of exit recorded.

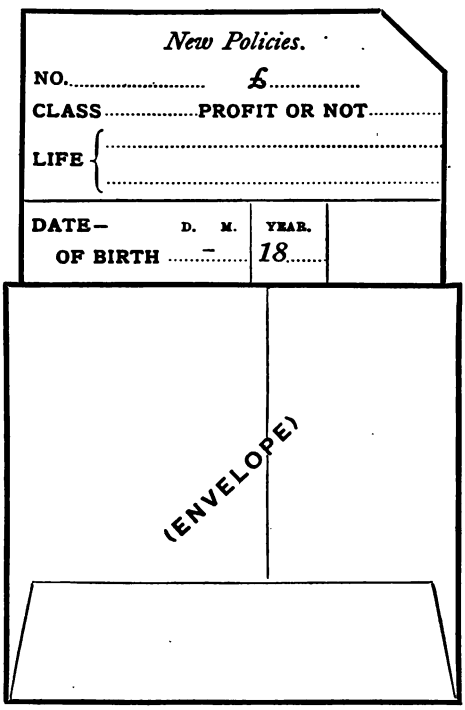
Endowment Assurance cases were treated as follows:—If the calendar year of exit was one wherein the life assured would attain an age which was a multiple of 5, then the birthday in that year was adopted as the date of exit; but if the calendar year corresponded with one which gave the assurance duration as a multiple of 5, then the policy anniversary was adopted as the date of exit.

All other cases of Terminations whereof the date of exit was defective, were dealt with upon the assumption of an even distribution through the calendar year, and the "cycles" used were such as would give the assumed nearest duration.

Appendix P.

METHODS ADOPTED FOR COLLOCATION OF DUPLICATES.

I. CHRONOLOGICAL SORTING.—The cards being already, for purposes of marking the Ages (*vide* Appendix D *supra*) in half-years of birth, were sorted within each year into strict order of day and month of birth. A comparison of the cases where the lives were born on the same day was then made, it being, of course, necessary for the operator to reduce the cards to an alphabetical order of surname, unless, as in the case of the years of birth embracing the largest number of cases, this had been carried into effect by other hands. The cards relating to persons of the same name, born on the same day, and wherein other particulars were not inconsistent, were placed in an envelope as Duplicates. The form of the envelope (see specimen below) was such as to take the lower half of the cards, leaving the name distinctly visible, and thus requiring no writing or marking upon the envelope.



Cards wherein the particulars were not in accord, but nevertheless left some *presumption of identity*, were set aside as *Doubtful Duplicates*. These cases included (i) those of identity of surname, with difference of Christian name, as revealing possible error in the transcription of Christian name, especially in cases of the same initial, *e.g.*, David and Daniel, or of more names on one card than on the other. (ii) Cases of a varied spelling of the surname, shewing possible corruption, *e.g.*, Burnes and Barnes. These

variations sometimes occurred even in the first letter, and the memory had to be kept well in use to bring together such varieties as Cridland and Pridland. (iii) Cases of apparent perversion of the order of names, or of omission of surnames (the last Christian name appearing in its place). (iv) Cases of identity of name and date of birth, but with other particulars inconsistent with their relating to the same life, *e.g.*, different dates of death, or death in one case whilst another policy appeared to be subsisting at a later date.

A preliminary enquiry was made, after a certain proportion of cases had been so dealt with, in order to obtain from the Offices some information which should form a statistical basis for guidance in dealing with these varieties of doubtful duplicates, and a range was given to the cases set aside, for this enquiry to cover names of greater and less frequency. Of those with different Christian names, many, of course, referred to different lives, of different antecedents, &c., but many cases were found, in reply to enquiries directed to this end, to refer to TWINS, whilst not a few shewed evidence of clerical error, either as to name, on correction of which the cards were found to refer to the same life, or as to date of birth, which removed all appearance of possible identity.

In many cases the day of the month of birth was not supplied, and in each month of each year the cases of this type were kept before the operator for comparison with the cards relating to the several days.

II. ALPHABETICAL SORTING.—A. *Male Lives.*—Within each year of birth, the cases where only one card appeared to subsist in respect of one life, and the packets of cards in their envelopes relating to Duplicates, were, in each class of Assurance, *in respect of each year of Birth*, next sorted into complete alphabetical order. In doing this, cases of titled persons, where the surname was sometimes given and sometimes the title, and where the title had changed, and cases also of other changes of name, were kept separate for comparison under each of the names so appearing. In comparison of the cards under this second sorting, a check was effected upon the previous collocation, and corrections of omission or wrongful inclusion made. Further new varieties of "Doubtful Duplicates" arose, being those wherein were brought together cards relating to persons of the same Christian names and surnames and born in the same year, but (i) either on different days of the same month; (ii) or on the same day of different months; (iii) or on different days of different months. In order that these varieties, as well as those derived from the previous sorting, might be dealt with in the most uniform and trustworthy manner, and seeing that a large number of juniors were employed upon the work, the plan was followed of setting aside uniformly all cases of the types mentioned, leaving to a smaller and selected body of seniors to discriminate between the cases (*a*) which from commonness of the name, and absence of other similarities, might reasonably be presumed to relate to different lives; (*b*) which from the evidence of the cards, might reasonably be assumed to relate to the same life; (*c*) where the information on the cards was not sufficiently definite to substantiate the identity, and the commonness of the name was not sufficient to justify presumption of non-identity. From the above sortings were omitted those cases where the data as to birth were

defective, which were mainly of two varieties: age at entry only given; and year of birth only given. As these latter were considered untrustworthy, being often only a "Valuation Year of Birth," all the varieties were treated homogeneously. Each variety was sorted into alphabetical order of surname, and the internal duplicates were brought together and encased in envelopes. An assumed mean year of birth (derived from the age at entry) was recorded upon the cards, and subsequently, the cards for each year of assumed birth were compared with the corresponding cards with full data for the three years of which the assumed year was the central one, and a large number of duplicates was thus discovered.

On account of the volume of the data, a complete alphabetical sorting would have been unsatisfactory, in view of the large number of cards relating to common names leading to questions of possible identity, *e.g.*, it may be roughly estimated that the experience included

11,200	cases of	Smith (say 2,000 John and William Smith)
4,600	,,	Brown
2,120	,,	Wright
2,100	,,	Moore

not to mention many other names, as Clark, Martin, &c. An experimental combination was however, made of a group of years, but it may be stated that the resultant additional number of evident duplicates so found was not appreciable. A very great source of difficulty in the course of the collocation of duplicates was found in the large number of cases of the same name, even if the name were not one of the most usual. In bringing together cases of the same name, if the policy emerged by death, it was possible in most instances to decide from the date of death whether the cards in reality referred to the same life, but in the case where one or both of the policies were withdrawn from observation by lapse or surrender at an earlier date, or where one or both policies were existing in 1893, clearly no case arose for any presumption of identity, so that it would of course be quite possible to identify cases of death without corresponding identification of cases not so eventuating. Furthermore, from the records on cards when dates of death and names (not very common) agreed entirely, and from some typical inquiries, it appeared that the date of exit in the case of deaths was not invariably the actual date of death, but was the nearest that the books of the contributing company were competent to supply, varying from the date of notification to the date of payment. The cases of defective date of exit where the year only was given, and that sometimes an Office year, in place of the true calendar year, as well as those cases where a mean date had been computed from the records and entered in the experience cards, formed a conspicuous feature of difficulty. All these cases of doubt gave rise to a large group of cases in regard to which inquiry might have to be made of the Offices for further particulars as to address and occupation for comparison. As such a course was quite impracticable on such a large scale, a series of rules, given below, was framed for dealing with such cases, and the carrying of these into effect was entrusted to seniors who had become well versed in the frequent recurrence of particular types of coincidence.

RULES FOR DEALING WITH "DOUBTFUL DUPLICATES" ARISING IN THE SUCCESSIVE STAGES OF THE COLLOCATION OF DUPLICATES.

WHOLE-LIFE AND ENDOWMENT ASSURANCES.
WITH AND WITHOUT PROFITS. MALE LIVES.

1. Throw out all cases of duplicates between Sections.

2. **Slight discrepancy in spelling** of surname or Christian name. Ignore, where other particulars afford strong probability of identity.

3. **Name, and day and year of birth agree. Months of birth differ.**
   (*a*) Assume identity only in those cases where the name is rare, or the dates of death substantially agree.
   (*b*) In other cases, assume non-identity.

4. **Name, and month and year of birth agree. Days of birth differ.**
   (*a*) Assume identity only in cases where the name is very rare, or the dates of death substantially agree.
   (*b*) In other cases, assume non-identity.

5. **Name and year of birth agree. Days and months of birth differ.**
   (*a*) Assume identity only in cases of very exceptional names, or in case of other very strong presumptive evidence of identity.
   (*b*) In other cases, assume non-identity.

6. **Date of birth and surname agree. Christian names differ.**
   (*a*) Where dates of death absolutely agree, assume identity.
   (*b*) Where dates of death differ by more than three months, assume non-identity.
   (*c*) Where the initials are the same and the name not very common, assume identity, unless other particulars furnish probability of non-identity.
   (*d*) Where the name is rare, but other particulars do not confirm identity, set aside for enquiry.

7. **Name agrees. Days, months, and years of death all differ.**
   (*a*) Where dates of death differ by more than three months; assume non-identity.
   (*b*) Where dates of death are within three months:—
      (*i*) In common names, assume identity—only if dates of birth are the same.
      (*ii*) In uncommon names, assume identity—if the whole particulars afford reasonable grounds.

8. Defective Data.

(*a*) Where the defective data as to birth or exit are not inconsistent with the particulars on other cards:—

Assume identity if the name is the same, and reasonable grounds exist.

(*b*) Where the defective data as to birth or exit are inconsistent with particulars on other cards:—

Assume non-identity — unless there exist very strong presumptions of identity.

9. In all the above Sections—

Set aside cases not covered by the rule, for enquiry of the Offices.

After application of the above rules, there still remained 2,110 cards to return to the Offices, relating to 784 cases of possible identity. Of these cards, 966, representing 391 cases, related to queries as between two or more cards contributed by the same Office, and 1,114, representing 393 cases, to those as between cards contributed by different Offices.

Upon return of the cards, the following results were observed:—

Inquiries as to apparent identities, referred to the same Office in 391 cases, succeeded in establishing identity in 288 cases.

Inquiries as to apparent identities, referred to different Offices in 393 cases, succeeded in establishing identity in 214 cases.

FEMALE LIVES.—The same general principles were followed in the collocation of Duplicates, with the following differences:—

1. Owing to the numerous cases of change of name by marriage and re-marriage, it was necessary to form separate alphabetical sortings, for comparison with the other cases of each of the names given. For this purpose, in the first or chronological sorting, the name alphabetically first was underlined, and in the alphabetical sorting the cards bearing more than one surname were sorted according to the name alphabetically first. In the course of the comparison of the cards in such alphabetical sorting, after the duplicates had been brought together, the cases of more than one surname were separated from the main body, and sorted according to the name alphabetically second, an additional stage being introduced for re-comparison of these cases, both internally, and also with the main body, for the discovery of further duplicates. The few cases with more than two names were also specially compared to the same end.

2. Owing to the small body of cards, it was practicable to arrange a complete alphabetical sorting, independently of the year of birth; and where cases seemed to point to possible identity of the lives, flagrant discrepancies in the dates of birth notwithstanding, the cards were set aside, for inquiry of the contributing Offices as to the correctness of the data.

Appendix Q.

RULES FOR DISTINCTIVE MARKING OF THE CARDS FOR THE ELIMINATION OF DUPLICATES (WITH NOTES THEREON).

WHOLE-LIFE AND ENDOWMENT ASSURANCES.
WITH AND WITHOUT PROFITS. MALE LIVES.

NOTE (A). Duplicates are only to be dealt with within their own sections taken separately, viz.:—

Old OP	New OP	Old EP	New EP
„ ON	„ ON	„ EN	„ EN

(B) Independent risks upon the same life are to be treated as though on different lives, whether (*a*) they occur before any of the cards have been marked, or (*b*) after the cards marked X (in accordance with the following rules) have been set aside.

(C) In the case of Terminations on the anniversary (or the day before) in 1893, or of Withdrawals on the day of the anniversary in 1893, alter the mode of exit to Existing.

OLD ASSURANCES.

RULE 1. If there be only one card upon any life in any particular section mark it S.A.

2. Arrange the cards upon the same life according to Age at Entry, and for each age at entry select one card, viz., the one having the earliest day and month of entry in the calendar year (such case coming first under observation in 1863). If there be two or more cards having the same day and month of entry, select the one having the longest duration. Mark all the other cards X in the left-hand top corner.

3. From among the unmarked cards select the earliest entrant under observation *—*i.e.*, where the day and month of entry is the earliest in the calendar year (irrespective of whether such be or be not the earliest assurant).

a. If such selected case further either (*a*) be Existing in 1893; or (*b*) record a Death; or (*c*) have a date of exit later than that of any other upon the same life in the section:—

(*i*) Mark the card S.A.

(*ii*) Mark the remaining cards S.

β. If such selected case does not fall within the above section *a*, place the cards in order of dates of entry in 1863.* Remove all cards (*a*) in which the dates of exit fall within the currency

* If there be two or more cards having the same day and month of entry, treat the one having the earliest year of entry as the earliest entrant under observation.

of an earlier policy, and (*b*) of which the dates of entry in 1863 are later than that of any card, which either is existing in 1893 or records a Death; so that the remaining cards shall form a series in the same order of exit as of entry in 1863.

(*i*) Mark the cards removed S.

(*ii*) Mark the first card of the series P.D.

(*iii*) Mark the remaining cards *P.D.* in the left-hand top corner, and A..... to the right of the duration of the policy; and place the cards so marked P.D. and *P.D.* in an envelope.

NEW ASSURANCES.

RULE 4. If there be only one card upon any life in any particular section, mark it S.H.

5. Arrange the cards upon the same life according to Age at Entry, and at each different age at entry select the earliest entrant, marking the remainder X.

NOTE.—If there be more than one card with the same day and month of entry, select the one with the longest duration.

6. From among the unmarked cards select the earliest assurant.

α. If such selected case further either (*a*) be Existing in 1893; or (*b*) record a Death; or (*c*) have a date of exit later than that of any other card upon the same life in the section:—

(*i*) Mark the card S.H.

(*ii*) Mark the remaining cards S.

β. If such selected case does not fall within the above section *α*, place the cards in order of age at entry. Remove all cards (*a*) in which the dates of exit fall within the currency of an earlier policy, and (*b*) of which the dates of entry are later than that of any card which either is existing in 1893 or records a Death; so that the remaining cards shall form a series in the same order of entry as of exit.

(*i*) Mark the cards removed S.

(*ii*) Mark the first card of the series P.D.

and (*iii*) Mark the remaining cards *P.D.* in the left-hand top corner, and A..... to the right of the duration of the policy; and place the cards so marked P.D. and *P.D.* in an envelope.

OLD AND NEW ASSURANCES.

RULE 7. Take the P.D. and *P.D.* cards, and in the space allotted on the stamp A....., enter the nearest integral duration on the card so marked, calculated from its own date of entry to the date of exit of the case immediately preceding in the same section.

Appendix R.

MINOR CLASSES OF ASSURANCE.

SELECT TABLES.

(*a*)

ASSURANCE DATA 1863-1893 — *Select Tables.* CLASS..... **M**

Assuring Age.....

Durn.	*Old.* P	*Old.* N	*New.* P	*New.* N	*Total.*
0.					
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10 & upw'ds					
Total					

AGGREGATE TABLES.

ASSURANCE DATA 1863-1893 — CLASS **M** (*b*)

Aggregate Tables.....

Age.	*Sectn.*	P	N	P+N
	OLD			
	NEW			
	O. & N.			
	OLD			
	NEW			
	O. & N.			
	OLD			
	NEW			
	O. & N.			
	OLD			
	NEW			
	O. & N.			
	OLD			
	NEW			
	O. & N.			

Cards of the form (*a*) were employed for the record of the numbers of entrants and emergents in the Minor Classes of Assurance. The particulars in the four Sections, Old and New Assurances, Participating and Non-participating, were separately recorded, and were totalled, on the cards. Separate cards were employed for Entrants, Deaths, and Existing; for Withdrawals, in four kinds, $W^{(1)}$, $W^{(4)}$, $W^{(7)}$, and $W^{(10)}$, and (deduced from these, where required), in two kinds, $W^{(a)}$, and $W^{(b)}$; also for Terminations, where required, in two kinds, $T^{(a)}$ and $T^{(b)}$.

In the classes of Contingent Assurances (Males), Temporary Assurances (Males), Joint Assurances (Males), and Joint Assurances (Females), the "Assuring age" at the head of the card included grouped ages, as follows:—0–17, 18–22, 23–27, ... 58–62, 63–end. In the classes of Whole Life Assurances with Limited Premiums, and with Ascending scale of Premiums, a card was employed for each separate Assuring age.

Abstract of Data cards of the form (*b*) were employed for the record of the data for Aggregate Tables in the following Minor Classes of Assurance:—Joint Assurances (Males); Joint Assurances (Females); Whole Life Assurances with Limited number of Premiums (Males); Whole Life Assurances, with Ascending scale of Premiums (Males). The particulars in the four Sections, "Old" and "New," Participating and Non-participating Assurances, were separately recorded, and were combined, on the cards. Each card included the record of five successive ages attained. Separate cards were employed for Entrants, Deaths, and Existing; also for Withdrawals, in four kinds, $W^{(1)}$, $W^{(4)}$, $W^{(7)}$, and $W^{(10)}$, with the deduced values $W^{(a)}$ and $W^{(b)}$, where required; and for Terminations, where required, in two kinds, $T^{(a)}$ and $T^{(b)}$.

[Cards of the form (*b*) were also conveniently employed for combining the data in respect of "Old" and "New" Participating and Non-Participating Assurances, for Whole-Life Assurances and Endowment Assurances.]

Appendix S.

AS TO DATA REFERRED BACK TO CONTRIBUTING OFFICES FOR EXAMINATION AND CORRECTION.

(1). *Queries to Offices.*—At the different stages of the work there came under notice various categories of cases which required special treatment, as well as those which it was necessary to refer to the Offices for correction or elucidation, or for addition to the particulars recorded upon the cards. These cases may be divided into two main classes of *Single Card Queries* and *Inter-card Queries.*

(2). SINGLE CARD QUERIES.—(i). These included, in the first place, cases where the data supplied on the card were defective in any respect. If these fell within the categories given as "Defective Data as to Birth" or "Defective Data as to Exit," already referred to in Appendices E and P, they were dealt with as thereunder described. In cases where the cards gave neither age, nor date of birth or baptism; date, but no mode of exit; or mode, but no date of exit; or, where the class was omitted, or was not in accordance with the provisions of clause (12) of the Instructions, it became of course necessary to return the cards to the Office for the supply of the necessary data. There also arose, under the sortings preparatory to the record of ages and durations, cases in which the dates were inconsistent, those of birth, entry, and exit, not being in chronological order, or otherwise impossible dates. These were necessarily referred to the Offices for correction. Cases where the dates of exit were prior to the anniversary in 1863, or the dates of entry later than 1892, were of course excluded: whilst those with dates of exit later than the anniversary in 1893 had to be altered to "Existing" on the anniversary in 1893. In the same way, in cases taken out before 1863, written on "New" cards, and cases taken out in 1863 or after, written on "Old" cards, particulars were re-written on cards of the correct form.

(3) (ii). The next type of "Single Card Queries" arose under the head of Collocation of Duplicates, and related to *Names.* In some cases, the name was either absent or incomplete (Christian name or initials omitted). In many cases, the name of what appeared to be a female life was entered on a white card (for male Lives) and *vice-versa*—or the sex was doubtful, in view of the name. In many cases, the cards were correctly written, the names being given in opposition to usual custom (*e.g.*, John to a female life—Alma to a male life, born on the day of the Battle, &c.). Where, however, it seemed evident that the particulars on a pink card (for female lives) related to a male life, the particulars were re-written on a white card. In the case, however, of particulars under the name of what appeared to be a female life being given on a white card, the cards were divided into two categories:—(*a*) If the assurance was taken out before the coming into force of the Married Women's Property Act, 1870, the particulars were re-written on a pink card. (*b*) If taken out subsequent to that Act, inquiry was made at the Office as to whether the name was correctly given, or whether the name

supplied was that of the beneficiary, and not that of the life assured. Under this heading 209 cases were referred to the Offices, 136 on white cards and 73 on pink cards, in which it was doubtful whether the sex of the life was correctly indicated by the colour of the card, of which in 107 and 45 cases respectively it was found that the sex, as indicated, was incorrect.

(4) (iii). In the course of the scrutiny of the cards there arose other forms of query as follows:—(1) Withdrawals at advanced ages, if under policies of long standing, were in many cases referred to the Offices for confirmation of the mode of exit. (2) In the Endowment Assurance class, deaths on the quinquennial birthday were referred to the Offices, and also cases near to the quinquennial birthdays 50, 55, 60, 65. Of these latter cases, 702 were referred to the Offices, and 21 proved to be incorrectly recorded as deaths. (3) Cases marked with mode of exit "T" in the Whole Life class, unless a note under the heading "Remarks" such as "Void by non-payment of extra premium," or as to change of class, satisfactorily explained the mode of exit. Many cases of "Suicide" were so treated by the Offices in writing up the cards, apparently because the full sum assured had not been paid. In these latter cases the mode of exit was altered to "Death." In all unexplained cases, inquiry was made of the Office for a satisfactory explanation. (4) Miscellaneous cases of exceptional data, such as assurances withdrawn after a very small interval, and others where the data seemed to cast suspicion on the correctness of the record. From inquiries of this type the necessity was discovered of the special treatment of the large and important body of "Transferred Office" cases, which are dealt with later on. (5) Small Paid-up Policies. In this case, the necessity of special treatment in all cases was revealed by the nature of some exceptional cases arising under queries between cards, and the treatment thereof is recorded below. (6) Cases where there had been an alteration in class were set aside, and afterwards re-examined, to see that the class at the head of the card was that under which the policy was originally taken out; and, where necessary, alteration was made to give effect to this. In addition to the above, there were many cases in which the class, as indicated, required explanation, or where the class was not one intended by the Instructions to be included in the assurance data. Under this type, 1,271 cards were excluded.

(5). INTER-CARD QUERIES; or queries arising as to the correctness of the data on one card, in view of the record of data upon other cards.—The majority of these cards necessarily arose under the stages of the "Collocation of Duplicates," and related to the records which presumably related to the same life. Many of these types have been already dealt with under the heading of "Doubtful Duplicates" (*see* Appendix P), and reference need only now be made to some other cases, requiring special treatment or inquiry of the Offices.

(6) (i). *Substituted Policies.*—(*a*) In many cases, independent cards had been written, where one policy had been substituted for another; and at the same time a note of such substitution was made on one or both of

the cards. As it was intended that only one card should appear, and that bearing the record of the whole period of risk, effect was given to such intention by modification of the earlier, and exclusion of the later, card. (*b*) In many cases, however, whilst there was no note of a substitution of one policy for another, it appeared that, at the time of cancelment of one policy, or within one month of such date, another policy was taken out for the same or a different amount in the *same* Office. As it would be the practice in many Offices to take such cases without fresh medical examination, if either the amount was not increased, or the mode of payment of premium not altered so as to increase the Office liability, it seemed clear that, even though the case might not be purely a substitution of one policy for another without modification of the premium, yet the risk was not one of "fresh selection" at the increased age. In these cases, when the sum assured, under the second policy, was the same as, or less than, that under the earlier policy, the risks were dealt with as follows:—Where the ratio of the premium to the sum assured appeared to be reduced (*e.g.*, where the change was from the ordinary Whole-Life assurance by uniform premiums to that by Ascending premiums), the two cases were treated as independent risks at different ages at date of assurance. Where, however, the ratio of the premium to the sum assured appeared to be the same or greater (*e.g.*, where the change was from the ordinary Whole-Life assurance by uniform premiums to that by Limited number of premiums) the cases were treated as not independent, but dealt with as follows:—If the policy anniversary of the earlier case was earlier in the calendar year than that of the later case, the experience of the earlier case was extended to include that of the later case—but not beyond the anniversary in 1893—the later card being excluded. If, however, the policy anniversary of the earlier case was later in the calendar year than that of the later case, then the later card was excluded, *without extension of the period of the risk in the earlier case.* By this means, the policy year method was retained, as though applied to the earlier case, and without introducing fractional periods of observation in the last year, if "Existing" in 1893 on the earlier anniversary of the later policy.

(7) (ii). *Lapsed Policies.*—Most of the cases of query as between cards, related to data which would be inconsistent if the cards related to the same life, and of this type especially were those arising from the application of varying regulations as to "lapse," "revival," or "non-forfeiture." Thus, in some such cases, the assurance having been retained upon the Company's books for a definite period as subject to revival, was not written off as lapsed until the close of such period, and the period of risk was similarly recorded on the experience cards. In collocating the duplicates, it appeared from the records of other cards (contributed by the same or another Office) that the life assured had died during the period in question, hence giving rise to a record of "Withdrawal" subsequent to that of "Death." In the case of duplicate risks, this was in part susceptible of correction; but in the case of unduplicated risks there was no possibility of correction of this source of error, which thus tended to include a certain record of exposure without record of the corresponding deaths.

(8) (iii). *Paid-up Policies.*—The same difficulty was met with in the case of Paid-up policies, cases being found which recorded a death under one policy, whilst a second policy which had been made paid-up for a reduced amount was recorded as existing at the anniversary in 1893. It was evident in these cases that a proportion of these Paid-up policies (possibly written up under "automatic non-forfeiture" regulations) had not matured into claims, *i.e.*, the deaths had not always been reported. To meet this difficulty, it was decided, in cases where the Paid-up policy was small in amount, both actually and relatively, to terminate the observation under the original assurance, at the date of the grant of the Paid-up policy. This was done in respect of all cases in which the Paid-up policy was for less than £20, and at the same time was less than 5 per cent. of the original sum assured.

(9) (iv). *Joint Life Assurances.*—Another stage of queries from the collocation of cards arose, in the case of Joint Life assurances, when bringing together the constituent cards relating to the different lives, assured under the same policy. A large number of queries as to inconsistency of the data and possible errors were dealt with by this means. The most frequent sources of error were:—(1) Reporting both lives "Died" on the same date, or both lives Terminated on the same date, when the card in respect of the life which died should alone have been marked "D" and the other "T"; (2) Cases of Last Survivor assurances written up as Joint Life assurances. Sometimes a footnote enabled the detection of these cases, and their exclusion from the observations, but in many instances they were only detected through the fact that more than one life was reported as "Died" and each case at a different date. From the number of these so discovered, it was evident that there may be a certain number of Last Survivor assurances still included, where the data would not exhibit the fact, *i.e.*, where the policy was "Withdrawn," or still "Existing" in 1893.

(10) (v). *Transferred Companies.*—In the course of inquiry upon certain points arising in connection with cards contributed to the experience, it became evident that the different Companies had followed varying practices, in supplying the data of the risks taken over from other Offices. It became necessary, therefore, to make direct inquiries of all those Companies contributing to the experience, which had taken over the business of other Offices. For this purpose the inquiry was limited to transfers of business between 1863 and 1893. A circular letter was then addressed to eighteen contributing Offices, in respect of twenty-seven Companies whose business had been transferred to them. In this letter, inquiry was made (1) whether the cards received from the Office included cards for policies issued by the transferred Company; and, if such were the case, (2) whether such cards were written up as (*a*) "Entering" at the date of transfer, or (*b*) "Entering" at the date of the original policy; and further, in the case (*b*), (3) whether cards were written for cases in the transferred Company which ceased to exist between 1863 and the date of transfer. The replies received were of five classes, as under:—

(A) Cards for transferred cases not written, and therefore excluded from the experience—16 transferred Offices.

(B) Cards written for transferred cases, as from original date of entry, but omitting the cases which ceased to exist prior to the transfer—5 transferred Offices.

(C) Cards written for transferred cases, but with date of transfer as date of entry—2 transferred Offices.

(D) Cards written for transferred cases, as from original date of entry, and also for cases which ceased to exist prior to the date of transfer—2 transferred Offices.

(E) One contributing Company, in the course of the inquiries with regard to two transferred Offices, stated that prior to 1863 they had absorbed a large number of other Companies whose registers they had used until 1866. In that year they commenced entering up new registers, embodying only those cases which then remained in force, the preparation of the new registers extending over a period of two years. In preparing the cards for the Mortality Experience, the Company had, in writing up the cards, supplied the original date of entry, but had omitted all cases which did not find place in such new registers—not only in respect of the Companies whose business had been taken over since 1863, but also in respect of nine Companies whose business had been transferred to them at earlier dates.

(11). In Classes A and D no difficulty arose, as the cards were either correctly written, or not supplied. In Class C the date given as that of "Entry" was not that of selection, and it was therefore desirable to exclude the cases. Of the two Offices, one had supplied only some 150 cards, so that it was not worth while to take any corrective steps. In the second Office, the cards relating thereto bore the name of the transferred Company in the "Remarks," so that it was possible to identify and exclude the cards (1,549 cases). Classes B and E were of the same type, resulting in giving periods of initial exposure, without the corresponding deaths. For three of the five Offices included in Class B, the Companies were able to supply the additional data, and thus rectify the error, furnishing some 2,500 further cards. In one Company, the cards numbered only about 150, and no corrective steps were taken. In the case of the fifth Company, transferred in 1864, the data for the years prior to transfer were not available, and it became necessary to modify the cards, as explained below. To assist in identifying the cards, it was considered expedient to obtain duplicates of the cards required, by preparing a second set from the Office books, facilities for which were afforded by the Company. By this means it was possible to trace the original cards to their place under the various sortings then current, so that they might be dealt with, as hereafter explained.

(12). In the case of the Company referred to under Class E, the cards affected by this error numbered some 12,000. In these cases, the cards bore special marks which enabled identification, and were separated from the main

body for examination, this operation being assisted by duplicate cards written up from the Office books. The cards relating to the transferred Offices contributed by the Company in Class E, and the Company, above referred to, included in Class B, were then dealt with as follows:—(1) "New" Policies, *i.e.*, those taken out after 1862, were excluded from observation. This was done so that the "New" experience might consist wholly of policies observed from entry. (2) "Old" Policies, which ceased to exist before their anniversary in 1869, were excluded from observation. (3) "Old" Policies in force upon their anniversary in 1869 were treated as coming under observation upon that date, in lieu of the anniversary in 1863; thus bringing them all under observation, uniformly, upon an anniversary subsequent to the latest date of true entry under observation.

NOTES ON THE

PRINCIPLES AND METHODS ADOPTED

IN THE

GRADUATION OF THE EXPERIENCE.

ANNUITANT EXPERIENCE,

SELECT TABLES.

MALES ($O^{[am]}$), FEMALES ($O^{[af]}$).

WHOLE-LIFE PARTICIPATING ASSURANCE EXPERIENCE:—

I. AGGREGATE TABLES—MALES—O^{M} AND $O^{M(5)}$.

II. SELECT TABLES—MALES—$O^{[M]}$.

BY

GEORGE F. HARDY, F.I.A.

NOTES ON THE PRINCIPLES AND METHODS

ADOPTED IN THE

GRADUATION OF THE EXPERIENCE.

(1). Having at the request of the Committee undertaken the graduation of the Mortality Tables, based upon the Annuitant Experience, and upon the Whole-Life Participating Assurance Experience (Males), I have drawn up the following memorandum as to the principles and methods employed.

The mortality tables in question are as follows :—

(1) Male Annuitant Experience.

(2) Female do.

(3) O^{M} Table, representing the aggregate experience of the Whole-Life Participating class for male lives.

(4) $O^{M(5)}$ Table, representing the aggregate experience of this class, excluding the first five years of assurance.

(5) $O^{[M]}$ Table, representing the aggregate of the various select tables compiled from the experience of this class for each age at entry.

It will be convenient to take these five tables in their order, stating, as briefly as is consistent with completeness, the methods adopted and the results arrived at in each case.

ANNUITANT EXPERIENCE, $O^{[am]}$ and $O^{[af]}$.

(2). The object of a graduation is something more than merely to remove the accidental irregularities in the ungraduated rates which are inherent in all statistical data. In the present instance, at any rate, the object is to provide a basis for the construction of financial tables which, while adhering closely to all important features of the original experience, shall at the same time be both safe and convenient in practical use. The uses which the graduated tables are to

subserve must therefore be borne in mind in any proposed treatment of the data.

(3). It has now been long recognised in respect to the mortality of annuitants or assurants that the period elapsed from the date of entry is a factor only less important than the age of the life, and the results of the new experience show that the length of time during which this factor is operative is greater than has generally been supposed. In dealing with the annuitant experience, the advanced ages at which lives continue to enter render it specially important to take account of the element of duration, as annuity values based upon an aggregate table would differ materially from the true values of annuities at entry.

(4). Owing, indeed, to the long period after entry during which the mortality rates are affected by the "self selection" of the annuitants, it is not sufficiently accurate to assume that after a period of five, or even ten, years the annuity values on select lives can safely be based upon an aggregate table. This is shown by the following comparison of the values of the ungraduated expectations upon select lives five and ten years respectively after entry with those deduced from the corresponding aggregate tables, excluding respectively the first five and ten years of duration.*

TABLE I.

Male Annuitants.

Comparison of Values of $e_{[x-5]+5}$, and of $e_{[x-10]+10}$, with Values by corresponding Aggregate Tables.

Groups of Ages	MEAN OF 10 VALUES OF e_x							
	Aggregate, excluding first 5 years	Select (after 5 Years)	Select Tables		Aggregate, excluding first 10 years	Select (after 10 Years)	Select Tables	
			+	−			+	−
50–59	16·61	16·39	...	·22	16·05	15·38	...	·67
55–64	13·87	14·16	·29	...	13·22	13·75	·53	...
60–69	11·24	11·38	·14	...	10·94	11·30	·36	...
65–74	8·70	8·79	·09	...	8·52	8·55	·03	...
70–79	6·51	6·61	·10	...	6·29	6·43	·14	...
75–84	4·77	5·17	·40	...	4·60	4·64	·04	...

*NOTE.—The extreme irregularity of mortality rates deduced from data classified with respect both to age and duration, makes it necessary to adopt some more certain and convenient method of measuring the duration of "selection" than a simple comparison of the ungraduated rates, and the most convenient method will, I think, be found to be by comparison of the values of e_x. As will be seen, extensive use has been made of this method of comparison.

TABLE II.

Female Annuitants.

Comparison of Values of $e_{[x-5]+5}$, and of $e_{[x-10]+10}$, with Values by corresponding Aggregate Tables.

Groups of Ages	MEAN OF 10 VALUES OF e_x							
	Aggregate, excluding first 5 years	Select (after 5 Years)	Select Tables		Aggregate, excluding first 10 years	Select (after 10 Years)	Select Tables	
			+	−			+	−
50–59	19·78	19·40	...	·38	19·47	19·59	·12	...
55–64	16·31	16·35	·04	...	16·04	15·86	...	·18
60–69	13·02	13·25	·23	...	12·82	12·93	·11	...
65–74	10·06	10·23	·17	...	9·93	10·21	·28	...
70–79	7·50	7·73	·23	...	7·42	7·53	·11	...
75–84	5·39	5·55	·16	...	5·36	5·47	·11	...

(5). It will be apparent from these figures that, if a junction were effected after five, or even after ten, years' duration, between the select tables and the aggregate, the expectations of life (and therefore, undoubtedly, the annuity values) at entry, would, for the more important ages, be under-estimated. As by far the most important function to be derived from the annuity tables is the value of the annuities at the date of entry, this would be a serious defect in the graduated mortality tables. At the same time, it appeared impracticable to publish, still more so to use, tables tracing the effect of selection for a greater period than ten years, and inconvenient to do so for longer than five or six years. This difficulty was overcome by replacing the aggregate mortality table, into which the select tables would otherwise have been merged at a given period from entry, by a *hypothetical* table, giving expectations of life and annuity values as nearly as possible in agreement with the values of these functions according to the ungraduated select tables five years after entry. This course had the advantages (1) that the values of the annuities according to the proposed table would accurately represent the values of annuities five years after entry, so far as they can be ascertained from the data; (2) they could be safely and conveniently employed for the valuation of annuities of more than five years from entry; and (3) as already stated, they would lead to the true annuity values at the date of entry.

(6). A brief examination of the unadjusted values of the expectations of life for the male annuitants brought out a point of considerable importance, the nature of which is indicated by the

figures in the following Table III, where the aforesaid values are compared with corresponding values by the H^M Table (Text-Book graduation) which latter, it must be understood, are used merely as a base line to throw into relief the features of the unadjusted experience:—

TABLE III.

Male Annuitants.

Comparison of Values of $e_{[x]}$ and $e_{[x]+5}$, with values of e_x and e_{x+5} (H^M Text-Book graduation).

Group of Entry Ages (1)	Mean of Five Values of		Difference (2)−(3) (4)	Mean of Five Values of		Difference (5)−(6) (7)
	$e_{[x]}$ Male (Annuitants) (2)	e_x (*Text-Book*) (3)		$e_{[x]+5}$ Male (Annuitants) (5)	e_{x+5} (*Text-Book*) (6)	
40–4	24·35	25·44	−1·09	20·08	21·86	−1·78
45–9	20·63	21·86	−1·23	17·11	18·42	−1·31
50–4	18·83	18·42	+0·41	15·67	15·16	+0·51
55–9	16·09	15·16	+0·93	12·64	12·15	+0·49
60–4	13·16	12·15	+1·01	10·11	9·45	+0·66
65–9	10·32	9·45	+0·87	7·48	7·12	+0·36
70–4	8·22	7·12	+1·10	5·74	5·16	+0·58
75–9	6·36	5·16	+1·20	(4·05)*	3·60	+0·45
80–4	4·32	3·60	+0·72	(2·73)*	2·39	+0·34

* Taken from Aggregate Table, excluding first five years from entry.

It is here evident that, whatever the cause, the vitality of the lives entering between 40 and 50 is abnormally low, a remark that applies in a lesser degree to lives entering between 50 and 55 (although the survivors of this latter group five years after entry appear to be average lives).

(7). It would, in my opinion, be improper to retain a feature of this nature in a graduated table intended as a basis for monetary tables, and accordingly the values of the expectations of life for ages 55 and upwards alone have been used as the basis for the male table.

(8). The practical advantage accruing from the use of Makeham's formula in the adjustment of mortality tables, advantages so well known that I need not insist upon them here, have led me to use it whenever this can be done without material distortion of the facts, and I have shown elsewhere* that the formula can be adapted to select tables by substituting for the usual formula for the force of mortality the formula

$$\mu_{[x]+t} = A + F(t) + [1 + \phi(t)]Bc^{x+t},$$

without loss of the valuable property that the values of annuities on

* *Journal of the Institute of Actuaries*, vol. xxxi, p. 359, and vol. xxxiii, p. 493.

two or more lives at different ages can be found from a table of such values for lives of equal age.

(9). A preliminary graduation of the aggregate tables, for both males and females, excluding the first five years after entry, showed that the male mortality from age 40 upwards could be well represented by Makeham's formula, while the female mortality could be fairly represented by a single curve at the older ages only, requiring below age 65 a supplementary curve, which, in order to retain the principle of "uniform seniority" as far as possible, was taken so that $l_x = l_x^{(1)} + l_x^{(2)}$ where both $l_x^{(1)}$ and $l_x^{(2)}$ followed Makeham's law with the same value of $\log c$ as in the male table.

(10). In determining the constants for these curves, and generally, for all the curves dealt with in this note, the principle adopted was that of equating to zero the sum of the deviations of the adjusted and unadjusted numbers (whether these numbers represented deaths or expectations of life), and also the sum of the accumulated deviations. This principle, so far as it goes, is of course equivalent to the method of "moments," employed by Professor Karl Pearson in fitting frequency curves to statistical data, and is one of great convenience in practice. As there were generally three or more constants to determine, higher moments might have been taken into account by equating, for example, the second summation of the accumulated deviations to zero. But it was found better to reserve a certain liberty of choice from among the various possible curves (which fulfil the condition of making the total of the deviations and accumulated deviations zero) in order to obtain a better general agreement throughout the curve, and to avoid anomalous values of the constants. Especially is this the case in the annuity tables, where constants determined mainly by the observations at the older ages are employed for the graduated rates of mortality at the earlier ages.

(11). Accordingly, three trial graduations of the male table were made for assumed values of $\log c$ equal to ·040, ·038 and ·036, and as the aggregate table (excluding the first five years from entry) is itself of some interest, I give here a table showing the constants corresponding to these three graduations, and a comparison of the actual and estimated deaths for quinquennial age groups. (*See* Table IV.)

(12). Table IV is of considerable interest, as showing how similar are the characteristics of the three tables, notwithstanding

Table IV.

Male Annuitant Experience, excluding first 5 years from Date of Purchase.

Ages	Actual Deaths	Mean Errors $\frac{2}{3}\sqrt{npq}$	Adjusted Deaths according to Makeham's Formula $\mu_x = A + Bc^x$.								
			(α)			(β)			(γ)		
			$\log c = \cdot 04$ $A = \cdot 01249$ $B = \cdot 0000766$	Errors		$\log c = \cdot 038$ $A = \cdot 00922$ $B = \cdot 0001146$	Errors		$\log c = \cdot 036$ $A = \cdot 00591$ $B = \cdot 0001711$	Errors	
				+	−		+	−		+	−
(1)	(2)	(3)	(4)			(5)			(6)		
40–4	3	± 1	6	3	...	5	2	...	4	1	...
45–9	7	2	12	5	...	10	3	...	9	2	...
50–4	26	4	24	...	2	22	...	4	21	...	5
55–9	67	6	59	...	8	57	...	10	54	...	13
60–4	118	9	123	5	...	122	4	...	121	3	...
65–9	235	12	252	17	...	255	20	...	257	22	...
70–4	449	16	448	...	1	453	4	...	461	12	...
75–9	603	19	576	...	27	583	...	20	589	...	14
80–4	527	17	517	...	10	518	...	9	519	...	8
85–9	317	12	318	1	...	315	...	2	310	...	7
90–4	94	6	99	5	...	96	2	...	93	...	1
95–9	14	2	25	11	...	23	9	...	22	8	...
100–4	2	1	3	1	...	3	1	...	2	...	...
Totals	2,462	107	2,462	+48	−48	2,462	+45	−45	2,462	+48	−48
				±96			±90			±96	

the wide differences in the values of their respective constants. It will be seen that on the whole the middle curve, where log c equals ·038, gives the best results. The first of the three curves, although the total deviations are well within the expected amount, is open to the fatal objection, owing to the large value of the constant A, of giving rates of mortality much too high at the younger ages. The third curve, on the other hand, overstates the mortality in the important period between ages 65 and 75, although, as tested by the third summations of the deviations, it is superior to the second of the three curves. By adopting as final values A=·00902; B=·0001149; log$c$=·038; the value of B (Male Table) was made equal to Bc (Female Table, First Series). It is unnecessary here to produce the corresponding figures of the female experience, as no mortality tables were constructed from these graduations, these aggregate tables being unsuitable for representing the ultimate mortality rates of the select tables.

(13). With respect to the hypothetical tables, based upon the expectations of life five years after entry, into which it was proposed to merge the select tables, it is not practicable to reproduce, within the limits available, the whole of the calculations by which the values of the constants were deduced from the unadjusted expectations of life. It will sufficiently elucidate the principle of the method to show in detail how the constants for the male table were obtained.

(14). From Table I it appears that from age 55-84, the values of e_x in column (3), are about equal to those in column (2) for ages ·47 years younger. Hence, taking A=·00902, B=·0001103 (=·0001149$c^{-\cdot 47}$) as first approximations to the values required, a table of curtate expectations of life was computed by means of a mortality table based upon these constants, together with the values of the functions $\frac{1}{100}\cdot\frac{de_x}{dA}$ and $\frac{de_x}{dx}$ (the latter giving by implication the values of $\frac{de_x}{dB}$), and the table of comparison given in columns (1) to (7) of Table V was thus obtained.

(15). The "weights" given in Table V were arrived at as follows. A fair estimate can be formed of the average errors in the values of the unadjusted expectations, and hence of the weight to be attached to the several values, by considering the average magnitude (irrespective of sign) of the second differences of these functions in various parts of the table, since these would be practically zero, but for the accidental errors in the successive values of e_x. The

TABLE V.

Male Annuitants.

Equations for determining Constants for Hypothetical Mortality Table representing Expectations of Life five years after purchase.

$\mu_x = A + Bc^x$ [Where $\log_{10} c = \cdot 038$; $A = \cdot 00902 - \delta A$; $B = \cdot 0001103 - \delta B$; whence $\mu_x = (\cdot 00902 - \delta A) + (\cdot 0001103 - \delta B)c^x = (\cdot 00902 - \delta A) + \cdot 0001103 c^{x - \delta x}$ nearly.]

Age Group	Mean of 5 Values of $e_{[x-5],+5}$		Coefficients of		Residuals	Weights of Observations							Central Age of Group
	First Graduation	Unadjusted Data	$100\delta A = -\frac{d}{100dA}e_x = m$	$\delta x = -\frac{d}{dx}e_x = n$	(3)−(2) = r	= w	wm	wn	w.r	Σwm	Σwn	Σw.r	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
55–59	15·56	15·67	1·69	·59	+·11	2·6	4·39	1·53	+ ·29	18·90	14·56	− 3·26	57
60–64	12·69	12·64	1·16	·55	−·05	3·4	3·94	1·87	− ·17	14·51	13·03	− 3·55	62
65–69	10·08	10·11	·77	·49	+·03	5·3	4·08	2·60	+ ·16	10·57	11·16	− 3·38	67
70–74	7·76	7·48	·48	·43	−·28	6·6	3·17	2·84	−1·85	6·49	8·56	− 3·54	72
75–79	5·78	5·74	·28	·36	−·04	6·5	1·82	2·34	− ·26	3·32	5·72	− 1·69	77
80–84	4·15	(4·05)*	·15	·29	−·10	6·5†	·98	1·89	− ·65	1·50	3·38	− 1·43	82
85–89	2·85	(2·73)*	·08	·23	−·12	6·5†	·52	1·49	− ·78	·52	1·49	− ·78	87
Totals ...	...	...	...	...	...	...	18·90	14·56	−3·26	55·81	57·90	−17·63	Totals

* Taken from the aggregate table, excluding the first five years from entry, the date at individual ages at entry being too scanty.

† Taken as equal to weight of group 75-79.

following, however, seems to be a more satisfactory method of estimating the (relative) mean errors in the unadjusted values of e_x. If the total number exposed to risk for a given age at entry $=n$, and the total number of deaths out of these exposures $=nq$, the average deviation from the mean number may be taken as $\pm \cdot 8\sqrt{nq(1-q)}$. This was expressed as a percentage of the total mortality nq. Tables were then formed, separately for the males and females (based upon a first rough Makeham adjustment in each case), showing the change in the various values of e_x, corresponding to a change of 1 per cent. in the mortality; and combining these latter quantities with the "average deviations" in the mortality as found above, a measure was obtained of the *relative weights* to be given to the several unadjusted values of e_x. The average errors given in Tables XII to XIV (pp. 144, 145) are the reciprocals of these weights, and are somewhat less than the true average errors.

(16). Comparing the values given in columns 4, 5 and 6, the sets of equations for determining the corrections to the constants, as given in columns (8) to (13), were obtained. The totals of these columns give the final equations for determining δA and δx, from which resulted the values $\delta A = \cdot 241$, whence $A = \cdot 00902 - \cdot 00241 = \cdot 00661$; and $\delta x = -\cdot 537$, which changes the value of B (corresponding to change of $\cdot 537$ years in age) to $Bc^{\cdot 537} = \cdot 0001103 c^{\cdot 537} = \cdot 0001156$. This value was subsequently changed to $B = 000115335$ as explained in par. (20).

(17). Substituting these values in the equations of condition in Table V, we have the following residuals :—

TABLE VI.

Male Annuitants.

Residuals in computing Expectations of Life for Hypothetical Table.

Central Age of Group (1)	$+\cdot 241wm - \cdot 537wn$ (2)	$w.r$ (3)	Second Residuals (4)	Σ Second Residuals (5)
57	+ ·24	+ ·29	+ ·05	·00
62	— ·05	— ·17	— ·12	—·05
67	— ·41	+ ·16	+ ·57	+·07
72	— ·76	—1·85	—1·09	—·50
77	— ·82	— ·26	+ ·56	+·59
82	— ·78	— ·65	+ ·13	+·03
87	— ·68	— ·78	— ·10	—·10
Totals	—3·26	—3·26	+1·31 —1·31	+·69 —·65

Cleared of the values of w (the weights attached to the original equations in Table V), the values of e_x corresponding to these constants are as under :—

TABLE VII.

Male Annuitants.

Comparison of Adjusted and Unadjusted Values of $e_{[x-5]+5}$

Central Age of Group	Mean of 5 Values of $e_{[x-5]+5}$		Errors	
	Unadjusted	Adjusted	+	−
57	15·67	15·65	...	·02
62	12·64	12·68	·04	...
67	10·11	10·00	...	·11
72	7·48	7·65	·17	...
77	5·74	5·65	...	·09
82	(4·05)*	4·03	...	·02
87	(2·73)*	2·75	·02	...
Average of all Groups	8·346	8·344	±·067	

* Taken from Aggregate Tables, excluding the 1st 5 years from entry.

(18). The above method leads, of course, to a first approximation only, as quantities of the second order are omitted, and a second approximation should therefore be now made. The effect, however, of such further approximation would only affect the values of e_x by unity in the second place of decimals, and the error will be found to be on the safe side. In the present instance it was rendered unnecessary from the fact that an arbitrary reduction was made in the value of A from ·00661 to ·0060 (corresponding to ·0026 in the constant portion of colog p_x) as otherwise the graduated rates of mortality at the younger ages in the male annuitant table would have materially exceeded the corresponding rates in the assurance experience, the data for which was published during the progress of the annuity graduation. This arbitrary change in the computed value of one of the constants was not made without due deliberation, and is justified on the ground of the smallness of the data for the younger ages at entry and by the undesirability of adopting upon such slender data, values of annuities for annuitant lives appreciably lower than for assured lives, seeing that it has in

recent years been not uncommon for annuities to be granted and assurances to be effected simultaneously upon the same individuals; and further by the fair agreement it yields from age 55 to 80 between the adjusted and unadjusted 3 per cent. annuity values at the date of entry, as will be seen by the comparison given in Table XV (p. 145).

(19). The female experience was dealt with on similar principles, except that, as in the aggregate table, it was found necessary to introduce a supplementary curve, in order satisfactorily to reproduce the values of the expectations for ages below 65. The average values of the expectations for the age groups 65–9, 70–4 . . . 85–9, having been duly weighted upon the principle above described, were employed to obtain the constants for the first curve representing the mortality for the older ages, the same value of log c=·038 being adopted as in the male table. To the resulting series, $l_x^{(1)}$, was added a supplementary series of the form $l_x^{(2)}=\kappa a^{x+t} l_{x+t}$ (where l_{x+t} is taken from the male table) in order to reproduce the values of the expectations for the age groups 40–4, 45–9, and onwards, the values of κ, a, and t being found by successive approximation by means of similarly formed and weighted equations of condition.

(20). As, in the graduation of the aggregate Tables, by a small modification of the value of B in the Male Table, its value was made equal to Bc (Female Table, First Series), whence resulted the relation for the older ages, where the supplementary series is insignificant,

μ_x (Female Table, 1st Series)=μ_{x-1}(Male Table)—Constant,

cologp_x(Female Table, 1st Series)=cologp_{x-1}(Male Table)—Constant.

It follows from this relation that the supplementary series above mentioned $l_x^{(2)}=\kappa a^{x+t} l_{x+t}$ may be formed from either the male l_x series, or from the first female series of $l_x^{(1)}$, suitably varying a and t. The value of t=24, was taken for convenience as an integer.

(21). The object of securing these relations between the male and female constants was to facilitate the calculation of joint life annuities, tables of annuities for two or more lives, male or female, either alone or in combination, being obtainable from a series of tables for corresponding combinations of lives all at equal ages. This point will be referred to later on.

(22). In determining the constants for the first five years after entry, the following objects were kept in view:—

(1) The production of a smooth juncture between the ultimate table and that representing the first 5 years after entry.

(2) The reproduction as closely as possible of the values of the unadjusted expectations at date of entry.

(3) Similarity in the modifications in the constant B for both male and female tables.

(4) The adoption of deductions to be made from the values of A and B in the ultimate tables to obtain their values for years 0, 1,.....4, such as would enable the values of $\mu_{[x]}$, $\mu_{[x]+1}$, &c., to be exactly computed.

(23). In these circumstances, although certain analytical methods were employed to obtain approximately the required modifications in the constants, the process was necessarily of a somewhat tentative character, and it will be sufficient here to state the values ultimately adopted (set out in Table VIII), as on the whole best fulfilling the conditions laid down.

(24). We have then, for the ultimate mortality table, in respect of durations of five years and upwards, the usual formulæ:—

$$\log_{10} l_x = \log_{10} k + x \log_{10} s + \log_{10} g . c^x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\operatorname{colog}_{10}(p_x) = -\Delta \log_{10} l_x = -\log_{10} s - (c-1)\log_{10} g . c^x = \alpha + \beta c^x \quad (2)$$

also $$\mu_x = -\frac{d}{dx}\log_e l_x = \frac{1}{M}\left(\alpha + \beta \frac{\log_e c}{c-1} \cdot c^x\right) = A + B c^x . \quad . \quad (3)$$

where M = the modulus $= \log_{10} e = \cdot 43429448$.

For durations of less than five years, the quantities α, β; A, B; and $\log_{10} k$, $\log_{10} g$; vary progressively with t, the duration from purchase, and the formulæ, corresponding to the above, become:—

$$\log_{10} l_{[x]+t} = (\log_{10} k - f_t) - (x+t)\alpha - \beta\left(\frac{1}{c-1} + \frac{\psi_t}{c^t}\right) c^{x+t}$$

$$= \log_{10} k_t + (x+t)\log_{10} s + \log_{10} g_t . \; c^{x+t} \quad . \quad . \quad . \quad . \quad (4)$$

$$\operatorname{colog}_{10}(p_{[x]+t}) = -\Delta_t \log_{10} l_{[x]+t} = (\alpha + \Delta f_t) + \beta\left(1 + \frac{\Delta \psi_t}{c^t}\right) c^{x+t}$$

$$= \alpha_t + \beta_t . c^{x+t} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

also $$\mu_{[x]+t} = -\frac{d}{dt}\log_e l_{[x]+t} = \frac{1}{M}\left[(\alpha + \frac{d}{dt} f_t) + \beta\left(\frac{\log_e c}{c-1} + \frac{1}{c^t} \cdot \frac{d}{dt}\psi_t\right) c^{x+t}\right.$$

$$= A_t + B_t . c^{x+t} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

It is also evident that

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\operatorname{colog}_{10}(p_{[x]+t}) = \operatorname{colog}_{10}(p_{x+t}) + \Delta f_t + \beta c^x \Delta \psi_t \quad . \quad . \quad . \quad (8)$$

$$\mu_{[x]+t} = \mu_{x+t} + \frac{1}{M}\left[\frac{d}{dt} f_t + \beta c^x \frac{d}{dt}\psi_t\right] \quad . \quad . \quad (9)$$

where these last equations (7), (8) and (9) are the forms naturally employed in the process of graduation, and equations (4), (5) and (6) are the forms most convenient for use in the actual construction of the mortality tables.

(25). The forms of the functions f_t and ψ_t respectively, both for the male table, and for the two separate series forming in combination the female table, are as follows:—

$$f_t = m[(5-t)^2 + (4-t)^2 - (1-t)^2] \quad . \quad . \quad . \quad . \quad (10)$$

$$\psi_t = n[(5-t)^2 + (4-t)^2 - (1-t)^2] \quad . \quad . \quad . \quad . \quad (11)$$

where the terms $(5-t)^2$, $(4-t)^2$, &c., in the square brackets are to be taken only for positive values of $(5-t)$, $(4-t)$, &c., and, of course, vanish, together with their differential coefficients, when $t=5$, $t=4$, &c.

For the male table, and for the second series of the female table, the value of $m = \cdot 000110$; for the first series of the female table $m = \cdot 000050$; the value of n throughout is $\cdot 0113 c^5 = \cdot 0175$. It follows that the successive values of α_t, β_t; A_t, B_t; $\log_{10} k_t$, $\log_{10} g_t$, employed in the construction of the graduated male table, differ from those employed in the construction of the first, and of the second, series constituting the graduated female table. The value of c, and the successive values of ψ_t, are, however, identical throughout. The appended Table VIII shows the actual values of the quantities employed in the graduation of the tables, for values of t from 0 to 4 inclusive; and also for the ultimate table.

(26). In the construction of the tables, the course followed was to compute the values of $\operatorname{colog}_{10}(p_x)$ and of $\operatorname{colog}_{10}(p_{[x]+t})$ by formulæ (2) and (5) given above, the radix being taken as $\log_{10} l_{[20]} = 5 \cdot 000000$. Tables IX and XI are appended, showing the values of $\mu_{[x]+t}$ and of μ_x for the male and female tables; also Table X, giving the fundamental values of $\operatorname{colog}_{10}(p_{[x]+t})$, and of $\operatorname{colog}_{10}(p_x)$, for the male table. (*See* pp. 138 to 143.)

TABLE VIII.

Graduation Functions.

Male Annuitant Experience.

t	A_t	B_t	a_t	β_t	$\log_{10} B_t$	$\log_{10} \beta_t$	$\log_{10} k_t$	$\log_{10} g_t$
0	·001 934 171	·000 064 712	·000 950	·000 033 109	$\bar{5}$·810 983	$\bar{5}$·519 944	5·055 558	—·000 618 263
1	·002 440 740	·000 080 548	·001 280	·000 039 756	$\bar{5}$·906 057	$\bar{5}$·599 407	5·057 208	—·000 596 801
2	·003 453 878	·000 094 087	·001 720	·000 045 425	$\bar{5}$·973 528	$\bar{5}$·657 296	5·058 528	—·000 583 227
3	·004 467 015	·000 105 601	·002 160	·000 049 528	$\bar{4}$·023 667	$\bar{5}$·694 849	5·059 408	—·000 575 983
4	·005 480 153	·000 112 362	·002 490	·000 051 700	$\bar{4}$·050 619	$\bar{5}$·713 495	5·059 848	—·000 573 106
5 or more	·005 986 721	·000 115 335	·002 600	·000 052 346	$\bar{4}$·061 960	$\bar{5}$·718 884	5·059 958	—·000 572 460

Female Annuitant Experience—*First Series.*

0	—·008 749 823	·000 059 290	—·003 750	·000 030 335	$\bar{5}$·772 978	$\bar{5}$·481 940	4·701 018	—·000 566 459
1	—·008 519 565	·000 073 799	—·003 600	·000 036 425	$\bar{5}$·868 052	$\bar{5}$·561 403	4·701 768	—·000 546 795
2	—·008 059 048	·000 086 203	—·003 400	·000 041 619	$\bar{5}$·935 524	$\bar{5}$·619 291	4·702 368	—·000 534 358
3	—·007 598 531	·000 096 753	—·003 200	·000 045 378	$\bar{5}$·985 663	$\bar{5}$·656 844	4·702 768	—·000 527 722
4	—·007 138 014	·000 102 947	—·003 050	·000 047 369	$\bar{4}$·012 615	$\bar{5}$·675 490	4·702 968	—·000 525 086
5 or more	—·006 907 755	·000 105 671	—·003 000	·000 047 960	$\bar{4}$·023 956	$\bar{5}$·680 879	4·703 018	—·000 524 495

Female Annuitant Experience—*Second Series.*

0	·014 068 795	·000 528 424	·006 220	·000 270 361	$\bar{4}$·722 983	$\bar{4}$·431 944	4·817 410	—·005 048 626
1	·014 575 364	·000 657 744	·006 550	·000 324 644	$\bar{4}$·818 057	$\bar{4}$·511 407	4·819 060	—·004 873 364
2	·015 588 501	·000 768 296	·006 990	·000 370 933	$\bar{4}$·885 528	$\bar{4}$·569 296	4·820 380	—·004 762 521
3	·016 601 639	·000 862 318	·007 430	·000 404 435	$\bar{4}$·935 667	$\bar{4}$·606 849	4·821 260	—·004 703 376
4	·017 614 776	·000 917 528	·007 760	·000 422 177	$\bar{4}$·962 619	$\bar{4}$·625 495	4·821 700	—·004 679 881
5 or more	·018 121 345	·000 941 803	·007 870	·000 427 448	$\bar{4}$·973 960	$\bar{4}$·630 884	4·821 810	—·004 674 610

ABSOLUTE CONSTANTS:—$c = 1{\cdot}091\,440\,4$; $\log_{10} c = {\cdot}038$; $\log_e c = {\cdot}087\,498\,2$; $\log_{10} s = -a$.

NOTE.—$(\mu_{[x]+t} - A_t)$ Male $= (\mu_{[x+1]+t} - A_t)$ Female 1st Series $= (\mu_{[x-24]+t} - A_t)$ Female 2nd Series.

(27). For the final female table, representing the numbers living, and the force of mortality, by $l^{(1)}$, $\mu^{(1)}$, under the *First* partial series; by $l^{(2)}$, $\mu^{(2)}$, under the *Second* partial series; and by l, μ, under the Final (combined) Table; we have :—

$$l_{[x]+t}=l^{(1)}_{[x]+t}+l^{(2)}_{[x]+t}$$

$$\mu_{[x]+t}=\frac{\mu^{(1)}_{[x]+t}\cdot l^{(1)}_{[x]+t}+\mu^{(2)}_{[x]+t}\cdot l^{(2)}_{[x]+t}}{l_{[x]+t}} \quad . \quad . \quad . \quad . \quad (12)$$

Similarly, for the ultimate female table :—

$$l_x=l^{(1)}_x+l^{(2)}_x$$

$$\mu_x=\frac{\mu^{(1)}_x\cdot l^{(1)}_x+\mu^{(2)}_x\cdot l^{(2)}_x}{l_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

It follows from these relations, that, in the case of a joint-life annuity on a male life (x) and a female life (y),

$$a_{xy}=\frac{a_{xy_1}+r_y.a_{xy_2}}{1+r_y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where y_1 and y_2 represent the mortality of the life (y) in respect of the first and second series of the female table, and $r_y=\frac{l_{y_2}}{l_{y_1}}$, the values of a_{xy_1} and a_{xy_2} for all ages of (x) and (y) being obtainable from tables of annuities at equal ages ww_1 and ww_2; and, if yz be two female lives,

$$a_{yz}=\frac{a_{y_1z_1}+r_ya_{y_2z_1}+r_za_{y_1z_2}+r_yr_za_{y_2z_2}}{(1+r_y)(1+r_z)} \quad . \quad . \quad . \quad . \quad (15)$$

If the annuity be on three lives (x) (y) (z), where (x) is a male life, and (y) and (z) are female, then

$$a_{xyz}=\frac{a_{xy_1z_1}+r_ya_{xy_2z_1}+r_za_{xy_1z_2}+r_yr_za_{xy_2z_2}}{(1+r_y)(1+r_z)} \quad . \quad . \quad . \quad . \quad (16)$$

These formulas are applicable to either select or non-select lives. In Table XIA (p. 166) are given the values of the ratio $r_{[y-t]+t}$ for all values of y, and for values of $t=0$, 1, 2, 3 and 4, and 5 or more.

TABLE IX.

Male Annuitants.

Select Tables—$0^{[am]}$.

Values of $\mu_{[x]+t}$ and of μ_{x+5}.

Age at Entry $[x]$	Years elapsed since Date of Purchase						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\mu_{[x]}$	$\mu_{[x]+1}$	$\mu_{[x]+2}$	$\mu_{[x]+3}$	$\mu_{[x]+4}$	μ_{x+5}	
20	·002 306 6	·002 946 6	·004 098 8	·005 257 1	·006 397 7	·007 014 6	25
21	·002 340 6	·002 992 9	·004 157 8	·005 329 3	·006 481 6	·007 108 6	26
22	·002 377 8	·003 043 4	·004 222 2	·005 408 2	·006 573 2	·007 211 2	27
23	·002 418 3	·003 098 5	·004 292 4	·005 494 2	·006 673 1	·007 323 2	28
24	·002 462 6	·003 158 6	·004 369 1	·005 588 2	·006 782 2	·007 445 4	29
25	·002 510 9	·003 224 3	·004 452 8	·005 690 7	·006 901 2	·007 578 8	30
26	·002 563 7	·003 295 9	·004 544 1	·005 802 6	·007 031 2	·007 724 4	31
27	·002 621 2	·003 374 1	·004 643 8	·005 924 7	·007 173 0	·007 883 3	32
28	·002 684 0	·003 459 5	·004 752 6	·006 058 0	·007 327 8	·008 056 7	33
29	·002 752 6	·003 552 6	·004 871 4	·006 203 5	·007 496 8	·008 246 0	34
30	·002 827 4	·003 654 3	·005 001 0	·006 362 3	·007 681 2	·008 452 5	35
31	·002 909 1	·003 765 3	·005 142 5	·006 535 6	·007 882 4	·008 678 0	36
32	·002 998 3	·003 886 4	·005 296 9	·006 724 7	·008 102 1	·008 924 1	37
33	·003 095 6	·004 018 6	·005 465 4	·006 931 2	·008 341 8	·009 192 7	38
34	·003 201 8	·004 162 8	·005 649 4	·007 156 5	·008 603 5	·009 485 9	39
35	·003 317 7	·004 320 3	·005 850 1	·007 402 4	·008 889 1	·009 805 8	40
36	·003 444 2	·004 492 2	·006 069 2	·007 670 8	·009 200 8	·010 155 0	41
37	·003 582 3	·004 679 8	·006 308 4	·007 963 8	·009 541 0	·010 536 2	42
38	·003 733 0	·004 884 5	·006 569 4	·008 283 5	·009 912 4	·010 952 2	43
39	·003 897 5	·005 108 0	·006 854 3	·008 632 5	·010 317 6	·011 406 2	44
40	·004 077 0	·005 351 8	·007 165 2	·009 013 4	·010 760 0	·011 901 8	45
41	·004 272 9	·005 618 0	·007 504 6	·009 429 1	·011 242 8	·012 442 7	46
42	·004 486 8	·005 908 6	·007 875 0	·009 882 9	·011 769 7	·013 033 0	47
43	·004 720 2	·006 225 7	·008 279 2	·010 378 1	·012 344 8	·013 677 3	48
44	·004 974 9	·006 571 8	·008 720 5	·010 918 6	·012 972 5	·014 380 6	49
45	·005 253 0	·006 949 5	·009 202 0	·011 508 6	·013 657 6	·015 148 1	50
46	·005 556 5	·007 361 8	·009 727 7	·012 152 4	·014 405 4	·015 985 8	51
47	·005 887 7	·007 811 8	·010 301 3	·012 855 2	·015 221 5	·016 900 1	52
48	·006 249 2	·008 302 9	·010 927 5	·013 622 2	·016 112 3	·017 898 1	53
49	·006 643 8	·008 838 9	·011 610 8	·014 459 4	·017 084 5	·018 987 2	54
50	·007 074 4	·009 424 0	·012 356 7	·015 373 1	·018 145 6	·020 176 0	55
51	·007 544 4	·010 062 5	·013 170 8	·016 370 3	·019 303 7	·021 473 5	56
52	·008 057 4	·010 759 5	·014 059 3	·017 458 8	·020 567 7	·022 889 6	57
53	·008 617 3	·011 520 1	·015 029 1	·018 646 7	·021 947 3	·024 435 2	58
54	·009 228 5	·012 350 4	·016 087 5	·019 943 3	·023 453 1	·026 122 1	59
55	·009 895 5	·013 256 5	·017 242 7	·021 358 5	·025 096 5	·027 963 3	60
56	·010 623 4	·014 245 5	·018 503 6	·022 903 1	·026 890 3	·029 972 9	61
57	·011 418 0	·015 324 9	·019 879 8	·024 588 9	·028 848 0	·032 166 2	62
58	·012 285 2	·016 503 1	·021 381 7	·026 428 8	·030 984 8	·034 560 0	63
59	·013 231 7	·017 788 9	·023 021 1	·028 437 0	·033 316 9	·037 172 8	64

TABLE IX (*continued*).

Male Annuitants.

Select Tables—$0^{(am)}$.

Values of $\mu_{[x]+t}$ and of μ_{x+5}.

Age at Entry [x]	Years elapsed since Date of Purchase.						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\mu_{[x]}$	$\mu_{[x]+1}$	$\mu_{[x]+2}$	$\mu_{[x]+3}$	$\mu_{[x]+4}$	μ_{x+5}	
60	·014 264 7	·019 192 4	·024 810 3	·030 628 8	·035 862 3	·040 024 4	65
61	·015 392 2	·020 724 1	·026 763 1	·033 021 1	·038 640 5	·043 136 9	66
62	·016 622 9	·022 396 0	·028 894 6	·035 632 0	·041 672 7	·046 533 9	67
63	·017 966 0	·024 220 7	·031 220 9	·038 481 8	·044 982 2	·050 241 5	68
64	·019 431 9	·026 212 2	·033 759 9	·041 592 1	·048 594 2	·054 288 2	69
65	·021 031 9	·028 385 9	·036 531 1	·044 986 8	·052 536 6	·058 704 9	70
66	·022 778 3	·030 758 4	·039 555 7	·048 692 0	·056 839 5	·063 525 5	71
67	·024 684 2	·033 347 7	·042 856 8	·052 735 9	·061 535 8	·068 786 8	72
68	·026 764 5	·036 173 9	·046 459 8	·057 149 7	·066 661 5	·074 529 3	73
69	·029 035 0	·039 258 4	·050 392 3	·061 967 0	·072 256 0	·080 796 8	74
70	·031 513 1	·042 625 1	·054 684 4	·067 224 8	·078 362 0	·087 637 5	75
71	·034 217 8	·046 299 5	·059 368 9	·072 963 4	·085 026 3	·095 103 7	76
72	·037 169 9	·050 310 0	·064 481 8	·079 226 7	·092 300 0	·103 252 6	77
73	·040 391 8	·054 687 2	·070 062 2	·086 062 7	·100 238 9	·112 146 6	78
74	·043 908 4	·059 464 6	·076 152 9	·093 523 9	·108 903 7	·121 853 9	79
75	·047 746 5	·064 678 9	·082 800 5	·101 667 3	·118 360 8	·132 448 8	80
76	·051 935 6	·070 370 0	·090 056 0	·110 555 3	·128 682 6	·144 012 5	81
77	·056 507 8	·076 581 4	·097 974 9	·120 256 0	·139 948 3	·156 633 6	82
78	·061 498 0	·083 360 9	·106 617 9	·130 843 8	·152 244 1	·170 408 8	83
79	·066 944 6	·090 760 2	·116 051 3	·142 399 8	·165 664 2	·185 443 6	84
80	·072 889 1	·098 836 2	·126 347 3	·155 012 4	·180 311 5	·201 853 2	85
81	·079 377 3	·107 650 6	·137 584 7	·168 778 3	·196 298 1	·219 763 3	86
82	·086 458 7	·117 271 0	·149 849 7	·183 802 9	·213 746 6	·239 311 2	87
83	·094 187 6	·127 771 1	·163 236 1	·200 201 5	·232 790 6	·260 646 5	88
84	·102 623 3	·139 231 4	·177 846 7	·218 099 5	·253 575 8	·283 932 6	89
85	·111 830 4	·151 739 6	·193 793 2	·237 634 2	·276 261 8	·309 348 1	90
86	·121 879 4	·165 391 5	·211 197 8	·258 955 1	·301 022 2	·337 087 5	91
87	·132 847 2	·180 291 8	·230 194 1	·282 225 5	·328 046 6	·367 363 5	92
88	·144 817 9	·196 554 6	·250 927 2	·307 623 9	·357 542 3	·400 408 0	93
89	·157 883 3	·214 304 4	·273 556 3	·335 344 6	·389 734 9	·436 474 0	94
90	·172 143 3	·233 677 3	·298 254 5	·365 600 1	·424 871 3	·475 837 8	95
91	·187 707 3	·254 821 6	·325 211 2	·398 622 2	·463 220 6	·518 801 2	96
92	·204 694 4	·277 899 5	·354 632 8	·434 664 0	·505 076 5	·565 693 1	97
93	·223 234 9	·303 087 5	·386 744 7	·474 001 4	·550 759 7	·616 872 9	98
94	·243 470 7	·330 578 8	·421 793 0	·516 935 7	·600 620 3	·672 732 6	99
95	·265 556 8	·360 583 8	·460 046 0	·563 796 1	·655 040 1	·733 700 0	100
96	·289 662 6	·393 332 5	·501 797 0	·614 941 3	·714 436 1	·800 242 3	101
97	·315 972 5	·429 075 8	·547 365 6	·670 763 3	·779 263 3	·872 869 3	102
98	·344 688 3	·468 087 5	·597 101 1	·731 689 7	·850 018 2	·952 137 4	103
99	·376 029 9	·510 666 4	·651 384 4	·798 187 1	·927 243 0	...	104

TABLE X.

Male Annuitants.

Select Tables—$0^{[am]}$.

Values of $\text{col}_{10}(p_{[x]+t})$ and of $\text{col}_{10}(p_{x+5})$.

Age at Entry [x]	Years elapsed since Date of Purchase						Age attained $x+5$
	0	1	2	3	4	5 or more	
	col $p_{[x]}$	col $p_{[x]+1}$	col $p_{[x]+2}$	col $p_{[x]+3}$	col $p_{[x]+4}$	col p_{x+5}	
20	·001 140 5	·001 529 7	·002 031 4	·002 530 6	·002 912 2	·003 066 5	25
21	·001 157 9	·001 552 5	·002 059 9	·002 564 4	·002 950 8	·003 109 2	26
22	·001 177 0	·001 577 5	·002 090 9	·002 601 4	·002 992 9	·003 155 8	27
23	·001 197 7	·001 604 6	·002 124 9	·002 641 8	·003 038 9	·003 206 6	28
24	·001 220 4	·001 634 3	·002 161 9	·002 685 8	·003 089 1	·003 262 0	29
25	·001 245 1	·001 666 7	·002 202 3	·002 733 9	·003 143 9	·003 322 6	30
26	·001 272 1	·001 702 1	·002 246 4	·002 786 4	·003 203 7	·003 388 7	31
27	·001 301 5	·001 740 7	·002 294 5	·002 843 7	·003 268 9	·003 460 8	32
28	·001 333 7	·001 782 8	·002 347 0	·002 906 2	·003 340 2	·003 539 5	33
29	·001 368 7	·001 828 8	·002 404 4	·002 974 4	·003 417 9	·003 625 4	34
30	·001 407 0	·001 879 0	·002 467 0	·003 048 9	·003 502 7	·003 719 1	35
31	·001 448 8	·001 933 7	·002 535 3	·003 130 2	·003 595 3	·003 821 5	36
32	·001 494 4	·001 993 5	·002 609 8	·003 218 9	·003 696 4	·003 933 2	37
33	·001 544 2	·002 058 8	·002 691 2	·003 315 7	·003 806 7	·004 055 1	38
34	·001 598 6	·002 130 0	·002 780 0	·003 421 4	·003 927 1	·004 188 1	39
35	·001 657 9	·002 207 7	·002 876 9	·003 536 7	·004 058 5	·004 333 3	40
36	·001 722 6	·002 292 5	·002 982 7	·003 662 6	·004 202 0	·004 491 8	41
37	·001 793 2	·002 385 1	·003 098 2	·003 800 0	·004 358 5	·004 664 8	42
38	·001 872 3	·002 486 2	·003 224 2	·003 950 0	·004 529 4	·004 853 6	43
39	·001 954 5	·002 596 5	·003 361 7	·004 113 7	·004 715 8	·005 059 7	44
40	·002 046 3	·002 716 8	·003 511 8	·004 292 3	·004 919 4	·005 284 6	45
41	·002 146 6	·002 848 2	·003 675 7	·004 487 3	·005 141 5	·005 530 1	46
42	·002 256 0	·002 991 6	·003 854 5	·004 700 1	·005 384 0	·005 798 0	47
43	·002 375 4	·003 148 1	·004 049 7	·004 932 4	·005 648 6	·006 090 5	48
44	·002 505 8	·003 319 0	·004 262 7	·005 185 9	·005 937 4	·006 409 6	49
45	·002 648 0	·003 505 4	·004 495 2	·005 462 5	·006 252 7	·006 758 0	50
46	·002 803 3	·003 708 9	·004 749 0	·005 764 5	·006 596 7	·007 138 2	51
47	·002 972 8	·003 931 0	·005 026 0	·006 094 1	·006 972 2	·007 553 2	52
48	·003 157 7	·004 173 4	·005 328 2	·006 453 9	·007 382 1	·008 006 1	53
49	·003 359 6	·004 438 0	·005 658 2	·006 846 5	·007 829 4	·008 500 4	54
50	·003 579 9	·004 726 7	·006 018 3	·007 275 0	·008 317 7	·009 040 0	55
51	·003 820 4	·005 041 9	·006 411 3	·007 742 8	·008 850 6	·009 628 8	56
52	·004 082 9	·005 385 9	·006 840 3	·008 253 2	·009 432 2	·010 271 6	57
53	·004 369 4	·005 761 3	·007 308 5	·008 810 4	·010 067 0	·010 973 1	58
54	·004 682 0	·006 171 1	·007 819 5	·009 418 5	·010 759 8	·011 738 7	59
55	·005 023 3	·006 618 4	·008 377 3	·010 082 3	·011 516 9	·012 574 3	60
56	·005 395 7	·007 106 5	·008 986 0	·010 806 7	·012 341 3	·013 486 4	61
57	·005 802 3	·007 639 3	·009 650 4	·011 597 3	·013 242 1	·014 481 8	62
58	·006 246 0	·008 220 8	·010 375 6	·012 460 3	·014 225 3	·015 568 3	63
59	·006 730 2	·008 855 4	·011 167 0	·013 402 1	·015 298 4	·016 754 1	64

TABLE X. (*continued*).

Male Annuitants.

Select Tables—$O^{[am]}$.

Values of $\text{col}_{10}(p_{[x]+t})$ and of $\text{col}_{10}(p_{x+5})$.

Age at Entry [x]	Years elapsed since Date of Purchase						Age attained $x+5$
	0	1	2	3	4	5 or more	
	col $p_{[x]}$	col $p_{[x]+1}$	col $p_{[x]+2}$	col $p_{[x]+3}$	col $p_{[x]+4}$	col p_{x+5}	
60	·007 258 8	·009 548 1	·012 030 9	·014 430 1	·016 469 6	·018 048 4	65
61	·007 835 6	·010 304 2	·012 973 7	·015 552 1	·017 747 9	·019 461 0	66
62	·008 465 3	·011 129 3	·014 002 8	·016 776 7	·019 143 1	·021 002 8	67
63	·009 152 5	·012 030 0	·015 125 9	·018 113 2	·020 665 8	·022 685 5	68
64	·009 902 5	·013 012 9	·016 351 7	·019 572 0	·022 327 9	·024 522 2	69
65	·010 721 1	·014 085 8	·017 689 7	·021 164 2	·024 141 8	·026 526 7	70
66	·011 614 6	·015 256 8	·019 149 9	·022 901 9	·026 121 7	·028 714 6	71
67	·012 589 8	·016 534 8	·020 743 7	·024 798 5	·028 282 6	·031 102 5	72
68	·013 654 1	·017 929 7	·022 483 3	·026 868 6	·030 641 0	·033 708 8	73
69	·014 815 8	·019 452 2	·024 381 9	·029 128 0	·033 215 2	·036 553 4	74
70	·016 083 7	·021 113 8	·026 454 1	·031 594 0	·036 024 7	·039 658 1	75
71	·017 467 5	·022 927 5	·028 715 8	·034 285 4	·039 091 1	·043 046 7	76
72	·018 977 9	·024 906 9	·031 184 3	·037 223 0	·042 438 0	·046 745 2	77
73	·020 626 3	·027 067 4	·033 878 5	·040 429 1	·046 090 8	·050 781 9	78
74	·022 425 5	·029 425 4	·036 819 1	·043 928 5	·050 077 7	·055 187 6	79
75	·024 389 3	·031 999 0	·040 028 5	·047 747 8	·054 429 1	·059 996 2	80
76	·026 532 6	·034 807 9	·043 531 5	·051 916 4	·059 178 4	·065 244 6	81
77	·028 871 8	·037 873 7	·047 354 7	·056 466 1	·064 362 1	·070 972 8	82
78	·031 425 0	·041 219 9	·051 527 6	·061 431 9	·070 019 7	·077 224 9	83
79	·034 211 7	·044 872 0	·056 082 0	·066 851 7	·076 194 6	·084 048 6	84
80	·037 253 1	·048 858 1	·061 052 9	·072 767 1	·082 934 2	·091 496 3	85
81	·040 572 7	·053 208 6	·066 478 3	·079 223 5	·090 290 0	·099 625 0	86
82	·044 195 8	·057 957 0	·072 399 9	·086 270 2	·098 318 5	·108 496 9	87
83	·048 150 2	·063 139 6	·078 862 8	·093 961 2	·107 081 1	·118 180 2	88
84	·052 466 2	·068 796 0	·085 916 8	·102 355 6	·116 644 9	·128 748 9	89
85	·057 176 9	·074 969 7	·093 615 8	·111 517 5	·127 083 3	·140 284 0	90
86	·062 318 3	·081 707 9	·102 018 8	·121 517 2	·138 476 1	·152 873 9	91
87	·067 929 8	·089 062 3	·111 190 1	·132 431 3	·150 910 7	·166 615 0	92
88	·074 054 5	·097 089 1	·121 200 1	·144 343 3	·164 482 4	·181 612 5	93
89	·080 739 2	·105 849 9	·132 125 4	·157 344 6	·179 295 0	·197 981 6	94
90	·088 035 1	·115 411 8	·144 049 7	·171 534 7	·195 462 1	·215 847 3	95
91	·095 998 2	·125 848 1	·157 064 4	·187 022 4	·213 107 5	·235 346 7	96
92	·104 689 5	·137 238 6	·171 269 2	·203 926 2	·232 366 5	·256 629 1	97
93	·114 175 4	·149 670 7	·186 772 8	·222 375 8	·253 386 4	·279 857 6	98
94	·124 528 8	·163 239 6	·203 694 1	·242 512 5	·276 328 5	·305 210 2	99
95	·135 828 9	·178 049 3	·222 162 7	·264 490 3	·301 368 4	·332 881 0	100
96	·148 162 3	·194 213 1	·242 320 0	·288 477 9	·328 697 8	·363 081 9	101
97	·161 623 4	·211 855 0	·264 320 5	·314 658 9	·358 526 5	·396 044 5	102
98	·176 315 4	·231 110 0	·288 332 8	·343 233 9	·391 082 6	·432 021 2	103
99	·192 350 9	·252 125 8	·314 540 8	·374 421 8	·426 615 6	...	104

TABLE XI.

Female Annuitants.

Select Tables—$0^{[a/]}$.

Values of $\mu_{[x]+t}$ and of μ_{x+5}.

Age at Entry $[x]$	Years elapsed since Date of Purchase.						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\mu_{[x]}$	$\mu_{[x]+1}$	$\mu_{[x]+2}$	$\mu_{[x]+3}$	$\mu_{[x]+4}$	μ_{x+5}	
20	·002 501 0	·003 214 6	·004 270 1	·005 317 7	·006 264 7	·006 778 7	25
21	·002 446 0	·003 195 6	·004 284 7	·005 365 6	·006 338 5	·006 873 6	26
22	·002 397 4	·003 185 5	·004 310 8	·005 427 4	·006 428 1	·006 985 6	27
23	·002 355 5	·003 184 9	·004 349 0	·005 503 8	·006 534 2	·007 115 5	28
24	·002 320 6	·003 194 2	·004 400 0	·005 595 7	·006 657 6	·007 263 9	29
25	·002 293 1	·003 214 1	·004 464 3	·005 703 8	·006 799 3	·007 432 3	30
26	·002 273 5	·003 245 0	·004 542 8	·005 828 8	·006 960 0	·007 620 8	31
27	·002 261 9	·003 287 4	·004 636 0	·005 971 4	·007 140 3	·007 830 2	32
28	·002 258 9	·003 341 8	·004 744 4	·006 132 4	·007 341 1	·008 061 4	33
29	·002 264 7	·003 408 7	·004 868 8	·006 312 3	·007 562 9	·008 314 8	34
30	·002 279 8	·003 488 7	·005 009 6	·006 511 8	·007 806 3	·008 590 9	35
31	·002 304 3	·003 581 9	·005 167 3	·006 731 3	·008 071 7	·008 890 1	36
32	·002 338 5	·003 688 9	·005 342 3	·006 971 3	·008 359 4	·009 212 3	37
33	·002 382 6	·003 810 0	·005 534 9	·007 232 0	·008 669 6	·009 557 8	38
34	·002 436 9	·003 945 2	·005 745 4	·007 513 5	·009 002 3	·009 926 1	39
35	·002 501 3	·004 094 8	·005 973 7	·007 815 7	·009 357 1	·010 316 9	40
36	·002 576 0	·004 258 6	·006 219 9	·008 138 5	·009 733 7	·010 729 3	41
37	·002 660 7	·004 436 7	·006 483 5	·008 481 4	·010 131 3	·011 162 5	42
38	·002 755 4	·004 628 7	·006 764 3	·008 843 7	·010 549 0	·011 615 1	43
39	·002 859 7	·004 834 2	·007 061 6	·009 224 4	·010 985 4	·012 085 5	44
40	·002 973 2	·005 052 6	·007 374 4	·009 622 4	·011 439 2	·012 572 0	45
41	·003 095 4	·005 283 1	·007 701 8	·010 036 2	·011 908 4	·013 072 5	46
42	·003 225 7	·005 524 8	·008 042 4	·010 464 3	·012 391 3	·013 585 1	47
43	·003 363 3	·005 776 6	·008 394 9	·010 904 8	·012 885 7	·014 107 6	48
44	·003 507 3	·006 037 3	·008 757 6	·011 355 8	·013 389 7	·014 638 1	49
45	·003 656 8	·006 305 6	·009 129 0	·011 815 5	·013 901 5	·015 175 1	50
46	·003 810 9	·006 580 2	·009 507 5	·012 282 3	·014 419 5	·015 717 9	51
47	·003 968 8	·006 859 9	·009 891 7	·012 754 8	·014 943 2	·016 266 6	52
48	·004 129 6	·007 143 8	·010 280 6	·013 232 5	·015 472 6	·016 822 9	53
49	·004 293 0	·007 431 3	·010 674 1	·013 715 7	·016 009 5	·017 390 2	54
50	·004 458 9	·007 722 5	·011 072 5	·014 206 1	·016 557 3	·017 974 4	55
51	·004 627 9	·008 018 3	·011 477 8	·014 707 0	·017 121 7	·018 584 1	56
52	·004 801 2	·008 320 7	·011 893 4	·015 224 1	·017 711 1	·019 231 1	57
53	·004 981 2	·008 633 2	·012 324 5	·015 765 5	·018 337 2	·019 931 0	58
54	·005 171 5	·008 961 1	·012 779 0	·016 342 3	·019 015 2	·020 703 2	59
55	·005 377 1	·009 311 6	·013 267 3	·016 969 0	·019 763 9	·021 570 9	60
56	·005 605 1	·009 694 5	·013 802 7	·017 663 6	·020 606 2	·022 561 3	61
57	·005 863 7	·010 121 7	·014 401 6	·018 447 7	·021 568 6	·023 705 1	62
58	·006 163 9	·010 608 1	·015 083 8	·019 346 4	·022 681 0	·025 035 6	63
59	·006 518 2	·011 171 0	·015 871 8	·020 387 7	·023 975 8	·026 588 3	64

TABLE XI (*continued*).

Female Annuitants.

Select Tables—$0^{[a/]}$.

Values of $\mu_{[x]+t}$ and of μ_{x+5}.

Age at Entry [x]	Years elapsed since Date of Purchase.						Age attained $x+5$
	0	1	2	3	4	5 or more	
	$\mu_{[x]}$	$\mu_{[x]+1}$	$\mu_{[x]+2}$	$\mu_{[x]+3}$	$\mu_{[x]+4}$	μ_{x+5}	
60	·006 941 3	·011 830 2	·016 790 9	·021 602 3	·025 487 6	·028 399 8	65
61	·007 449 3	·012 607 5	·017 868 3	·023 022 6	·027 251 9	·030 514 3	66
62	·008 060 0	·013 526 1	·019 132 7	·024 681 8	·029 304 2	·032 943 6	67
63	·008 791 9	·014 610 3	·020 613 4	·026 613 0	·031 679 0	·035 745 3	68
64	·009 663 9	·015 884 6	·022 339 3	·028 848 3	·034 409 2	·038 943 4	69
65	·010 694 8	·017 372 9	·024 338 6	·031 418 4	·037 525 6	·042 567 2	70
66	·011 902 9	·019 098 4	·026 637 8	·034 351 9	·041 057 0	·046 644 8	71
67	·013 305 2	·021 082 5	·029 261 5	·037 675 4	·045 030 4	·051 203 0	72
68	·014 917 6	·023 345 3	·032 232 9	·041 414 1	·049 472 0	·056 269 2	73
69	·016 755 0	·025 905 6	·035 573 7	·045 592 5	·054 408 4	·061 872 6	74
70	·018 830 9	·028 781 1	·039 305 3	·050 235 3	·059 868 0	·068 045 5	75
71	·021 158 5	·031 989 3	·043 449 4	·055 369 2	·065 882 2	·074 824 8	76
72	·023 751 3	·035 548 4	·048 029 5	·061 023 7	·072 487 2	·082 253 4	77
73	·026 623 4	·039 478 7	·053 072 1	·067 232 8	·079 724 5	·090 380 5	78
74	·029 790 9	·043 802 6	·058 607 6	·074 035 4	·087 642 4	·099 262 8	79
75	·033 272 1	·048 546 4	·064 671 0	·081 477 8	·096 295 8	·108 959 4	80
76	·037 062 5	·053 718 6	·071 287 6	·089 601 7	·105 742 6	·119 554 3	81
77	·041 251 6	·059 409 7	·078 543 0	·098 489 8	·116 064 4	·131 118 0	82
78	·045 823 8	·065 621 1	·086 462 0	·108 190 5	·127 330 1	·143 739 1	83
79	·050 814 0	·072 400 6	·095 105 0	·118 778 3	·139 625 9	·157 514 3	84
80	·056 260 6	·079 799 9	·104 538 4	·130 334 3	·153 046 0	·172 549 1	85
81	·062 205 1	·087 875 9	·114 834 4	·142 946 9	·167 693 3	·188 958 7	86
82	·068 693 3	·096 690 3	·126 071 8	·156 712 8	·183 679 9	·206 868 8	87
83	·075 774 7	·106 310 7	·138 336 8	·171 737 4	·201 128 4	·226 416 7	88
84	·083 503 7	·116 810 8	·151 723 2	·188 136 0	·220 172 4	·247 752 0	89
85	·091 939 3	·128 271 1	·166 333 8	·206 034 0	·240 957 6	·271 038 1	90
86	·101 146 4	·140 779 3	·182 280 3	·225 568 7	·263 643 6	·296 453 6	91
87	·111 195 4	·154 431 2	·199 684 9	·246 889 6	·288 404 0	·324 193 0	92
88	·122 163 2	·169 331 5	·218 681 2	·270 160 0	·315 428 4	·354 469 0	93
89	·134 133 9	·185 594 3	·239 414 3	·295 558 4	·344 924 1	·387 513 5	94
90	·147 199 3	·203 344 1	·262 043 4	·323 279 1	·377 116 7	·423 579 5	95
91	·161 459 3	·222 717 0	·286 741 6	·353 534 6	·412 253 1	·462 943 3	96
92	·177 023 3	·243 861 3	·313 698 3	·386 556 7	·450 602 4	·505 906 7	97
93	·194 010 4	·266 939 2	·343 119 9	·422 598 5	·492 458 3	·552 798 6	98
94	·212 550 9	·292 127 2	·375 231 8	·461 935 9	·538 141 5	·603 978 4	99
95	·232 786 7	·319 618 5	·410 280 1	·504 870 2	·588 002 1	·659 838 1	100
96	·254 872 8	·349 623 5	·448 533 1	·551 730 6	·642 421 9	·720 805 5	101
97	·278 978 6	·382 372 2	·490 284 1	·602 875 8	·701 817 9	·787 347 8	102
98	·305 288 5	·418 115 5	·535 852 7	·658 697 8	·766 645 1	·859 974 8	103
99	·334 004 3	·457 127 2	·585 588 2	·719 624 2	·837 400 0	...	104

NOTE.—In the final table, the rates of mortality given by the formula at the younger ages were slightly reduced, to avoid rates decreasing with the age. These changes only slightly modified the values of $l_{[x]+t}$ for values of $x=20$ to 24.

(28). The following tables show the value of the graduated expectations of life, as compared with the unadjusted value of these functions, both at the date of entry and after 5 years; Table XII for the original graduation of the male experience, Table XIII for that ultimately adopted, (the value of the constant a in the formula for $\text{colog}\, p_x$ being reduced to ·00260), and Table XIV for the female experience. It will be seen that, except that the values of the expectations in the second graduation of the male table are (owing to the arbitrary reduction just referred to) somewhat in excess on the whole of the adjusted values, there is in both tables a very fair agreement with the original data, the average deviations being considerably below the estimated amount.

TABLE XII.

Male Annuitants. Select Tables.
Unadjusted and Adjusted Expectations.
First Graduation (α=·00287).

Age	Mean of Five Values $e_{[x-5]+5}$			Errors		Mean of Five Values $e_{[x]}$			Errors	
	Unadjusted		Adjusted	+	−	Unadjusted		Adjusted	+	−
57	15·67	±·38	15·66	...	·01	16·09	±·29	16·05	...	·04
62	12·64	±·30	12·69	·05	...	13·16	±·20	13·10	...	·06
67	10·11	±·19	10·01	...	·10	10·32	±·15	10·45	·13	...
72	7·48	±·15	7·66	·18	...	8·22	±·14	8·14	...	·08
77	5·74	±·15	5·67	...	·07	6·36	±·16	6·17	...	·19
82	*(4·05)	±·15	4·04	...	·01	4·32	±·19	4·57	·25	...
87	*(2·73)	±·15	2·77	·04	...	...	...	...	...	...

TABLE XIII.

Male Annuitants. Select Tables.
Unadjusted and Adjusted Expectations.
Second Graduation (α=·00260).

Age	Mean of Five Values $e_{[x-5]+5}$			Errors		Mean of Five Values $e_{[x]}$			Errors	
	Unadjusted		Adjusted	+	−	Unadjusted		Adjusted	+	−
57	15·67	±·38	15·77	·10	...	16·09	±·29	16·15	·06	...
62	12·64	±·30	12·77	·13	...	13·16	±·20	13·17	·01	...
67	10·11	±·19	10·06	...	·05	10·32	±·15	10·50	·18	...
72	7·48	±·15	7·69	·21	...	8·22	±·14	8·16	...	·06
77	5·74	±·15	5·69	...	·05	6·36	±·16	6·19	...	·17
82	*(4·05)	±·15	4·05	...	...	4·32	±·19	4·57	·25	...
87	*(2·73)	±·15	2·77	·04	...	...	...	...	...	...

* Taken from Aggregate Table excluding 1st 5 years from entry.

TABLE XIV.

Female Annuitants. Select Tables.
Unadjusted and Adjusted Expectations.

Age	Mean of Five Values $e_{[x-5]+5}$			Errors		Mean of Five Values $e_{[x]}$			Errors	
	Unadjusted		Adjusted	+	−	Unadjusted		Adjusted	+	−
42	...	...	...	...	...	28·55	±·66	28·29	...	·26
47	24·42	±·45	24·48	·06	...	24·71	±·41	24·95	·24	...
52	21·28	±·32	21·21	...	·07	21·78	±·25	21·69	...	·09
57	17·90	±·23	17·99	·09	...	18·66	±·17	18·45	...	·21
62	14·92	±·16	14·78	...	·14	15·08	±·11	15·22	·14	...
67	11·57	±·11	11·67	·10	...	12·13	±·09	12·12	...	·01
72	8·89	±·09	8·87	...	·02	9·49	±·10	9·34	...	·15
77	6·58	±·10	6·50	...	·08	6·74	±·11	7·00	·26	...
82	4·53	±·12	4·60	·07	...	5·00	±·16	5·13	·13	...
87	*(3·14)	±·12	3·14	...	...	...	...	...	...	...

* Taken from Aggregate Table excluding 1st 5 years from entry.

(As the values of $e_{[x-5]+5}$ for ages 47 to 57 are somewhat below the values of e_x according to the Aggregate Table, excluding first five years from entry, the means of the respective values $e_{[x-5]+5}$ and e_x have been substituted at those ages.)

(29). It will be interesting, finally, to give a comparison between the unadjusted annuity values obtained by grouping the facts for five successive ages at entry, as already published for quinquennial ages in the volume of unadjusted data, and the values corresponding to the finally adjusted tables; and this is done in the following table:—

TABLE XV.

Annuitant Experience.
Comparative Table of Select Annuity Values at 3 per cent. Interest.

Age at Purchase	Male Lives				Female Lives			
	Values obtained by grouping facts for 5 Ages at Purchase	Values from Adjusted Tables	Errors in Adjustment +	Errors in Adjustment −	Values obtained by grouping facts for 5 Ages at Purchase	Values from Adjusted Tables	Errors in Adjustment +	Errors in Adjustment −
	(a)	(b)	(b) − (a)		(a′)	(b′)	(b′) − (a′)	
40	...	...	...	...	18·18	18·26	·08	...
45	...	...	...	...	16·99	16·93	...	·06
50	...	...	...	...	15·59	15·51	...	·08
55	12·67	12·66	...	·01	13·97	13·97	...	...
60	10·86	10·88	·02	...	12·16	12·23	·07	...
65	9·03	9·12	·09	...	10·28	10·33	·05	...
70	7·31	7·44	·13	...	8·47	8·41	...	·06
75	6·10	5·90	...	·20	6·64	6·61	...	·08
80	4·53	4·54	·01	...	4·94	5·05	·11	...
Average	8·417	8·423	·006	...	11·912	11·922	·010	...

WHOLE-LIFE PARTICIPATING EXPERIENCE.

MALE LIVES.

(30). With respect to the long duration of the effect of selection, and the difficulty of effecting a satisfactory junction between the mortality rates of the select experience and the corresponding aggregate table after so short a period as five years, the assurance experience presents similar features to that of the annuitants.

Here, however, it is obviously important that the table representing the ultimate mortality after the effect of selection is presumed to have worn off should not over-estimate the annuity values. In other words, it is most important that we should have a safe table for the purpose of valuations, and it is of somewhat less importance that the graduated table should represent precisely the true rates of premium at the date of entry, provided that these are not under-estimated, and that there is a fair agreement during the principal entry ages.

(31). The following Table XVI will be found very instructive as to the question of the effect and duration of selection. As already stated, I think the expectations for various periods after entry form the most satisfactory basis of comparison, where a rapid view of the effect of selection is desired.

TABLE XVI.

Whole-Life Participating Assurance Experience—Males.

Mean values of ungraduated curtate Expectations for 5 consecutive ages.

Grouped Ages	Select Tables: Individual Ages at Entry						Select Tables: Combined Ages at Entry		Truncated Aggregate Table	
	$e_{[x]}$	$e_{[x-5]+5}$	$e_{[x-10]+10}$	$e_{[x-15]+15}$	$e_{[x-20]+20}$	$e_{[x-25]+25}$	$e_x^{[5]}$	$e_x^{[10]}$	$e_x^{(5)}$	$e_x^{(10)}$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
20–24	40·91	...	...	...	...	...	40·72	40·27	40·59	40·50
25–29	37·57	36·88	...	...	...	...	36·99	36·76	36·80	36·65
30–34	33·56	33·43	33·17	...	...	...	33·25	33·06	33·06	32·93
35–39	29·95	29·45	29·65	29·54	...	...	29·54	29·40	29·37	29·27
40–44	26·36	25·87	25·74	25·97	25·95	...	25·85	25·74	25·70	25·62
45–49	22·62	22·33	22·15	22·16	22·42	22·38	22·26	22·20	22·13	22·08
50–54	19·38	18·73	18·76	18·68	18·73	19·05	18·79	18·75	18·67	18·64
55–59	16·28	15·70	15·35	15·44	15·44	15·47	15·51	15·48	15·42	15·39
60–64	13·38	12·74	12·57	12·24	12·41	12·42	12·46	12·44	12·38	12·37
65–69	10·56	10·07	9·74	9·73	9·55	9·65	9·72	9·71	9·66	9·66
70–74	8·97	7·72	7·62	7·32	7·28	7·19	7·36	7·35	7·32	7·32
75–79	...	...	5·69	5·55	5·36	5·31	5·40	5·39	5·37	5·36
80–84	...	...	...	4·37	4·06	3·72	3·84	3·84	3·82	3·82
85–89	...	...	...	...	3·03	3·11	2·74	2·74	2·71	2·71

(32). A comparison of these expectations will bring out the following points, which can only be briefly touched upon :—

(i). The age at entry in certain cases materially affects the mortality throughout the whole of life. For example, lives entering between 20 and 24, and in still more marked manner those entering between 25 and 29, show better rates of mortality at all ages passed through than the average. Twenty-five years after entry the average expectation of life $e_{[27]+25}(=19{\cdot}05)$ is distinctly higher than that of a life of corresponding present age having entered only five years previously $e_{[47]+5}(=18{\cdot}73)$.

(ii). The aggregate tables excluding either the first 5 or first 10 years of assurance (what have been termed the "Truncated Aggregate Tables") give lower expectations than the corresponding select table for combined ages at entry. This is, of course, due to the further exclusion of duplicate cases, in combining the experience for all ages at entry. It has the effect, however, of rendering it extremely difficult, at however remote a period after entry, to satisfactorily join on the mortality of the select tables for individual ages at entry, with the mortality of the corresponding truncated aggregate table—although a junction can be fairly well effected with the corresponding select table (combined ages at entry) after 10 years, and no doubt perfectly well after about 15 years from the date of assurance.

(33). In these circumstances, after much discussion and experiment, the Committee came to the decision to publish graduated tables representing—

AGGREGATE DATA :—

(*a*). An O^{M} Table, all ages at entry combined.

(*b*). An $O^{M(5)}$ Table, all ages at entry combined, but excluding the first five years of assurance.

SELECT DATA :—

(*c*). An $O^{[M]}$ (or Select) Table formed from the entire whole-life participating experience, distinguishing each age at entry for the first ten years of assurance, the individual mortality tables after ten years running into a table formed by the aggregation of the whole of the select data excluding the first ten years of assurance.

It may, I think, be said that the Committee in arriving at this decision recognised that there was a demand among actuaries, on

the one hand, for tables that would correspond to the now familiar combination of the H^M and $H^{M(5)}$ Tables, and, on the other hand, for Select Tables which would represent a more thorough analysis of the data, and in particular would afford a satisfactory basis for the calculation of the true "risk" premiums at the date of entry.

I. Aggregate Data—O^M & $O^{M(5)}$.

(34). With respect to the first two Tables, the O^M and $O^{M(5)}$, I must confess that I regarded the latter as by far the more important, and I adhere to this view, although it may not be generally shared. I propose to deal first, therefore, with the $O^{M(5)}$ Table.

(35). Makeham's formula in the present instance represents very fairly the general features of the experience. This will be seen from the following comparison (Table XVII) of the actual deaths with the numbers resulting from the adjusted rates of mortality. The bulk of the table, from age 25 to 79, lies very close to the original facts. After age 85, the constants employed in constructing the table somewhat exaggerate the mortality, and similarly below age 25. Neither of these features appears to me to be of great importance. It is possible that the rates of mortality in the original table are somewhat under-estimated at the extreme ages. An examination of cards representing lives at risk over 95 years of age showed that some of these exposures were fictitious, and simply due to the omission on the cards of the date and cause of exit. With such an enormous mass of entries, it was inevitable that a few such omissions should take place, and, while these would have very little effect upon the numbers exposed to risk in the body of the table, where these numbers amounted to tens of thousands at each age, they might at the extreme ages become of sufficient importance to unduly swell the exposed to risk, and so under-estimate the rates of mortality. In any case, I think that the observed rates of mortality above 90 need have but little weight attached to them, and, in determining the best constants for the table, I have taken the data between ages 20 and 89 only. It will be seen from Table XVII, that on the average the deviations of the adjusted from the unadjusted table are within the probable errors of observation.

The sum of the accumulated deviations is not accurately zero, as to produce this result a still smaller value of the constant A would have been necessary (viz., ·005866), the value being already slightly below that of the male annuitant table.

TABLE XVII.

Whole-Life Participating Assurance Experience—Males.

Aggregate Table, excluding first **FIVE** *Years' Experience.*

Combined "Old" and "New" Assurances.

$A = \cdot 005888861$ $B = \cdot 000103794$ $\mu_x = A + Bc^x$ Log $c = \cdot 039$

$\alpha = \cdot 002557500$ $\beta = \cdot 000047163$ Colog $p_x = \alpha + \beta c^x$

Group of Ages.	Expected Deaths.	Actual Deaths.	Deviations.		Expected Deviation.
			+	−	±
15–9	18	10	8	...	3
20–4	136	122	14	...	9
25–9	949	924	25	...	24
30–4	3,136	3,072	64	...	44
35–9	5,683	5,689	...	6	60
40–4	7,981	8,152	...	171	72
45–9	10,277	10,257	20	...	81
50–4	12,613	12,620	...	7	89
55–9	14,921	14,903	18	...	97
60–4	16,808	16,618	190	...	101
65–9	17,448	17,455	...	7	103
70–4	15,929	16,042	...	113	98
75–9	12,147	12,172	...	25	83
80–4	7,207	7,317	...	110	63
85–9	2,970	2,865	105	...	38
Total ...	128,223	128,218	444	439	±965
			± 883		

(36). A useful comparison of the graduated and ungraduated tables can also be made by means of the expectations of life. In Table XVIII, the mean expectations of life for quinary groups of ages are given according to the unadjusted 5-year aggregate table, and according to the graduated table, and for the sake of comparison according to the $H^{M(5)}$ Table. It will be seen that the first two series between ages 25 and 80 are practically identical, and that such differences as exist are extremely minute as compared with the differences between the $H^{M(5)}$ and the present tables.

(37). In the determination of the constants, the principle of moments, already referred to, was employed by comparing the summation and double summation of the graduated and actual deaths arranged in quinary groups. Three trial values of the constant log c, equal to ·038, ·039, and ·040 being employed, it was found that the intermediate value ·039 gave the most satisfactory result, and that no appreciable advantage was gained by taking this constant to

a larger number of decimals. In the first instance the expected deaths by the graduated tables were computed by taking the deaths at age x divided by the exposures in the middle of the year, as equal to $\mu_{x+\frac{1}{2}}$. This method was convenient, and sufficient for a first approximation; the expected deaths were, however, subsequently computed accurately by the adjusted values of q_x.

TABLE XVIII.

Whole-Life Participating Assurance Experience—Males.

Aggregate Table, excluding first* FIVE *Years' Experience.

Comparison of the Expectations of Life. (Mean of Five Values.)

Group of Ages	New Experience		Deviations A−U		$H^{M(5)}$	Differences $O^{M(5)} - H^{M(5)}$	
	Unadjusted	Adjusted $O^{M(5)}$	+	−		+	−
20–4	40·59	40·43	...	·16	38·45	1·98	...
25–9	36·80	36·77	...	·03	35·39	1·38	...
30–4	33·06	33·07	·01	...	32·04	1·03	...
35–9	29·37	29·38	·01	...	28·53	·85	...
40–4	25·70	25·71	·01	...	25·02	·69	...
45–9	22·13	22·12	...	·01	21·55	·57	...
50–4	18·67	18·67	...	...	18·20	·47	...
55–9	15·42	15·40	...	·02	15·02	·38	...
60–4	12·38	12·38	...	...	12·05	·33	...
65–9	9·66	9·68	·02	...	9·41	·27	...
70–4	7·32	7·32	...	...	7·05	·27	...
75–9	5·37	5·35	...	·02	5·15	·20	...
80–4	3·82	3·76	...	·06	3·67	·09	...

Average Deviation Ages 25–79, irrespective of sign, $= \pm$ ·012, or, with signs, $-$ ·0027.

In Table XIX are given the values of the graduation constants adopted, and their logarithms.

TABLE XIX.

Graduation Constants.—$O^{M(5)}$ Table.

Constant	Value	Common Logarithm	Napierian Logarithm
k	114 157·6	$5\cdot057\ 504\ 7$	$11\cdot645\ 334\ 9$
s	·994 128 7	$\bar{1}\cdot997\ 442\ 5$	$\bar{1}\cdot994\ 111\ 1$
g	·998 844 9	$\bar{1}\cdot999\ 498\ 0$	$\bar{1}\cdot998\ 844\ 2$
c	1·093 956 4	$0\cdot039\ 000\ 0$	$0\cdot089\ 800\ 8$
A	·005 888 861	$\bar{3}\cdot770\ 031\ 3$	$\bar{6}\cdot865\ 307\ 3$
B	·000 103 794	$\bar{4}\cdot016\ 170\ 9$	$\bar{10}\cdot826\ 894\ 5$
α	·002 557 500	$\bar{3}\cdot407\ 815\ 6$	$\bar{6}\cdot031\ 274\ 8$
β	·000 047 163	$\bar{5}\cdot673\ 601\ 4$	$\bar{10}\cdot038\ 099\ 1$

(38). In the adjustment of the O^M Table, the use of Makeham's formula alone was not practicable; not only did the facts not lend themselves very satisfactorily to the formula, but it was evidently desirable, for the sake of logical consistency, that the later portion of the table above age 85, where the facts were identical with those of the $O^{M(5)}$ data, should also be identical with the graduated $O^{M(5)}$ curve. The graduated O^M Table was accordingly built up on the basis of the graduated $O^{M(5)}$, by the addition of a double frequency-curve to the formula representing the function $\Delta \operatorname{colog}_{10}(p_x)$, the relation between this function in the two Tables being as follows :—

$$\Delta \operatorname{col}_{10}(p_x)^{O^M} = \Delta \operatorname{col}_{10}(p_x)^{O^{M(5)}} + \phi_x \quad . \quad . \quad . \quad . \quad (17)$$

$$\operatorname{col}_{10}(p_x)^{O^M} = \operatorname{col}_{10}(p_x)^{O^{M(5)}} - \Sigma_x^{\omega} \phi_x \quad . \quad . \quad . \quad . \quad (18)$$

where $\phi_x = \cdot 000050400 e^{-\cdot 0032 \log_e 10 (29-x)^2} + \cdot 000011385 e^{-\cdot 0060 \log_e 10 (66 \cdot 5 - x)^2}$ (19)

$$= \cdot 000050400 (10)^{-\cdot 0032 (29-x)^2} + \cdot 000011385 (10)^{-\cdot 0060 (66 \cdot 5 - x)^2} \quad (20)^*$$

(39). The values of the constants in formula (20) were obtained in the following manner:—The deviations between the unadjusted values of colog p_x for the O^M and $O^{M(5)}$ Tables having been set out graphically, and a smooth curve drawn through the unadjusted values, it was clear from the nature of the curve that a double frequency-curve of the above form with suitable co-efficients would well represent the observations. Approximate values of the constants were found by considering first the form of the curve at the older ages, and after this had been satisfactorily represented by the second term on the right-hand side of equation (18), by then dealing with the residual series. As, for reasons already stated, it was necessary to abandon Makeham's formula pure and simple, the only object in view was to obtain a perfectly smooth curve, and to represent as nearly as possible the ungraduated facts.

(40). Table XX gives a comparison of the graduated and ungraduated deaths under the full aggregate O^M Table, from which it will be seen that there is a sufficiently close agreement throughout between ages 10 and 89.

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* The fundamental values for the O^M Table were computed to six decimal places, by the formula given on page 1 of the published Tables, the results being practically identical with those given in Table XXI.

TABLE XX.

Whole-Life Participating Assurance Experience—Males.

Full Aggregate Table.

Same constants as in experience excluding first five years, with Supplementary Curve.

Group of Ages	Expected Deaths	Actual Deaths	Deviation		Expected Deviation	Accumulated Deviation	
			+	−	±	+	−
10–4	11	10	1	...	3	...	...
15–9	94	97	...	3	8	...	1
20–4	766	806	...	40	22	2	...
25–9	2,687	2,615	72	...	41	42	...
30–4	5,202	5,202	...	...	58	...	30
35–9	7,538	7,557	...	19	69	...	30
40–4	9,600	9,731	...	131	78	...	11
45–9	11,611	11,526	85	...	85	120	...
50–4	13,651	13,670	...	19	93	35	...
55–9	15,620	15,594	26	...	99	54	...
60–4	17,268	17,093	175	...	103	28	...
65–9	17,697	17,677	2	...	104	...	147
70–4	16,033	16,150	...	117	98	...	149
75–9	12,170	12,197	...	27	83	...	32
80–4	7,208	7,317	...	109	63	...	5
85–9	2,969	2,865	104	...	38	104	...
Total ...	140,107	140,107	465	465	±1,045	385	405
				±930			

(41). In the appended Table XXI the values of $\text{colog}_{10}(p_x)$ forming the fundamental basis of the O^{M} and $O^{M(5)}$ Tables respectively, are set out at every age, to seven places of decimals, together with the difference between the cologarithms of the probabilities.

II. Select Data—$O^{[M]}$.

(42). The graduation of the Whole-Life Participating Select Tables was the most important part of the present work, as these tables represent most nearly, and in the most completely analyzed form, the features of the original experience.

(43). As already stated, the duration of the effect of selection is very considerable, but a comparison of columns (4) and (9) in Table XVI (p.146) will show that on the whole it is nearly exhausted at the end of ten years, and there is obviously a great convenience in not analyzing the select mortality for a longer period after entry.

TABLE XXI.

British Offices Life Tables, 1893.

Values of col $_{10}(p_x) - 0^{M}$ & $0^{M(5)}$.

Age x	$0^{M(5)}$ col $_{10}(p_x)$	Deduction $= \Sigma_x^{\infty} \phi_x$	0^{M} col $_{10}(p_x)$ (2)−(3)	Age x	$0^{M(5)}$ col $_{10}(p_x)$	Deduction $= \Sigma_x^{\infty} \phi_x$	0^{M} col $_{10}(p_x)$ (2)−(3)
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
10	·002 673 3	·001 203 0	·001 470 3	60	·012 875 6	·000 150 8	·012 724 8
11	·002 684 2	·001 199 5	·001 484 7	61	·013 845 1	·000 144 4	·013 700 7
12	·002 696 1	·001 194 8	·001 501 3	62	·014 905 6	·000 136 9	·014 768 7
13	·002 709 1	·001 188 8	·001 520 3	63	·016 065 8	·000 128 3	·015 937 5
14	·002 723 3	·001 181 2	·001 542 1	64	·017 335 0	·000 118 7	·017 216 3
15	·002 738 9	·001 171 6	·001 567 3	65	·018 723 4	·000 108 2	·018 615 2
16	·002 755 9	·001 159 7	·001 596 2	66	·020 242 3	·000 097 2	·020 145 1
17	·002 774 6	·001 145 2	·001 629 4	67	·021 903 9	·000 085 8	·021 818 1
18	·002 795 0	·001 127 7	·001 667 3	68	·023 721 6	·000 074 5	·023 647 1
19	·002 817 3	·001 107 1	·001 710 2	69	·025 710 1	·000 063 5	·025 646 6
20	·002 841 7	·001 083 0	·001 758 7	70	·027 885 4	·000 053 0	·027 832 4
21	·002 868 4	·001 055 2	·001 813 2	71	·030 265 2	·000 043 4	·030 221 8
22	·002 897 6	·001 023 8	·001 873 8	72	·032 868 5	·000 034 8	·032 833 7
23	·002 929 5	·000 988 6	·001 940 9	73	·035 716 4	·000 027 3	·035 689 1
24	·002 964 5	·000 950 0	·002 014 5	74	·038 831 9	·000 020 9	·038 811 0
25	·003 002 7	·000 908 1	·002 094 6	75	·042 240 1	·000 015 7	·042 224 4
26	·003 044 6	·000 863 3	·002 181 3	76	·045 968 5	·000 011 5	·045 957 0
27	·003 090 3	·000 816 1	·002 274 2	77	·050 047 3	·000 008 2	·050 039 1
28	·003 140 4	·000 767 2	·002 373 2	78	·054 509 2	·000 005 8	·054 503 4
29	·003 195 2	·000 717 1	·002 478 1	79	·059 390 4	·000 003 9	·059 386 5
30	·003 255 1	·000 666 7	·002 588 4	80	·064 730 2	·000 002 6	·064 727 6
31	·003 320 6	·000 616 8	·002 703 8	81	·070 571 8	·000 001 7	·070 570 1
32	·003 392 3	·000 567 9	·002 824 4	82	·076 962 1	·000 001 1	·076 961 0
33	·003 470 8	·000 520 7	·002 950 1	83	·083 952 9	·000 000 7	·083 952 2
34	·003 556 6	·000 475 9	·003 080 7	84	·091 600 5	·000 000 4	·091 600 1
35	·003 650 4	·000 434 0	·003 216 4	85	·099 966 7	·000 000 2	·099 966 5
36	·003 753 1	·000 395 3	·003 357 8	86	·109 118 9	·000 000 1	·109 118 8
37	·003 865 5	·000 360 2	·003 505 3	87	·119 131 0	·000 000 1	·119 130 9
38	·003 988 4	·000 328 7	·003 659 7	88	·130 083 9	...	·130 083 9
39	·004 122 8	·000 301 0	·003 821 8	89	·142 065 8	...	·142 065 8
40	·004 269 9	·000 276 9	·003 993 0	90	·155 173 5	...	·155 173 5
41	·004 430 8	·000 256 2	·004 184 6	91	·169 512 7	...	·169 512 7
42	·004 606 8	·000 238 8	·004 368 0	92	·185 199 2	...	·185 199 2
43	·004 799 3	·000 224 3	·004 575 0	93	·202 359 6	...	·202 359 6
44	·005 009 9	·000 212 4	·004 797 5	94	·221 132 3	...	·221 132 3
45	·005 240 4	·000 202 7	·005 037 7	95	·241 668 8	...	·241 668 8
46	·005 492 4	·000 195 1	·005 297 3	96	·264 134 8	...	·264 134 8
47	·005 768 2	·000 189 1	·005 579 1	97	·288 711 6	...	·288 711 6
48	·006 069 9	·000 184 4	·005 885 5	98	·315 597 6	...	·315 597 6
49	·006 399 9	·000 180 7	·006 219 2	99	·345 009 8	...	·345 009 8
50	·006 760 9	·000 177 9	·006 583 0	100	·377 185 3	...	·377 185 3
51	·007 155 8	·000 175 7	·006 980 1	101	·412 384 0	...	·412 384 0
52	·007 587 9	·000 173 9	·007 414 0	102	·450 889 8	...	·450 889 8
53	·008 060 5	·000 172 2	·007 888 3	103	·493 013 5	...	·493 013 5
54	·008 577 5	·000 170 6	·008 406 9	104	·539 095 0	...	·539 095 0
55	·009 143 2	·000 168 8	·008 974 4	105	·589 506 1	...	·589 506 1
56	·009 761 9	·000 166 6	·009 595 3	...	...	...	...
57	·010 438 8	·000 163 9	·010 274 9	...	...	...	...
58	·011 179 3	·000 160 4	·011 018 9	...	...	...	...
59	·011 989 4	·000 156 1	·011 833 3	...	...	...	...

TABLE XXII.

Whole-Life Participating Assurance Experience—Male Lives.

Excluding the first **TEN** *Years' Experience.*

Males[10]. **Select Data.**

Age (1)	Exposed to Risk (2)	Died (3)	Rate of Mortality (4)	Curtate Expectation of Life (5)	Age (1)	Exposed to Risk (2)	Died (3)	Rate of Mortality (4)	Curtate Expectation of Life (5)
20	84	2	·02381	41·188	65	101,099	4,177	·04132	10·750
21	169	0	·00000	41·193	66	94,968	4,206	·04429	10·213
22	292	4	·01370	40·193	67	88,748	4,309	·04855	9·687
23	457	3	·00656	39·751	68	82,215	4,456	·05420	9·181
24	747	6	·00803	39·014	69	75,812	4,280	·05646	8·707
25	1,212	5	·00413	38·330	70	69,664	4,323	·06206	8·228
26	1,895	12	·00633	37·488	71	63,398	4,213	·06645	7·772
27	2,837	21	·00740	36·727	72	57,167	4,281	·07489	7·326
28	4,099	31	·00756	36·001	73	51,152	4,031	·07880	6·919
29	5,946	44	·00740	35·275	74	45,266	3,883	·08578	6·511
30	8,730	75	·00859	34·538	75	39,808	3,637	·09136	6·121
31	13,702	85	·00620	33·838	76	34,815	3,512	·10088	5·737
32	20,034	154	·00769	33·049	77	29,959	3,212	·10721	5·381
33	27,986	235	·00840	32·305	78	25,691	3,035	·11813	5·027
34	37,177	315	·00847	31·578	79	21,704	2,764	·12735	4·700
35	47,860	433	·00905	30·848	80	18,177	2,551	·14034	4·386
36	58,706	537	·00915	30·130	81	14,944	2,223	·14876	4·102
37	69,286	653	·00942	29·408	82	12,083	1,987	·16445	3·819
38	80,005	706	·00882	28·688	83	9,711	1,684	·17341	3·570
39	90,419	790	·00874	27·943	84	7,680	1,558	·20286	3·319
40	101,070	1,016	·01005	27·189	85	5,853	1,162	·19853	3·164
41	110,136	1,141	·01036	26·466	86	4,505	930	·20644	2·948
42	118,500	1,249	·01054	25·743	87	3,400	788	·23176	2·715
43	125,838	1,422	·01130	25·017	88	2,493	602	·24148	2·534
44	132,459	1,633	·01233	24·303	89	1,808	464	·25664	2·340
45	138,275	1,684	·01218	23·606	90	1,249	328	·26261	2·148
46	143,054	1,834	·01282	22·897	91	866	246	·28406	1·913
47	146,685	1,903	·01297	22·195	92	606	195	·32178	1·673
48	149,210	2,153	·01443	21·486	93	393	154	·39186	1·466
49	150,870	2,187	·01450	20·801	94	226	93	·41150	1·411
50	152,521	2,380	·01560	20·107	95	125	57	·45600	1·398
51	152,829	2,478	·01621	19·425	96	65	15	·23077	1·569
52	152,392	2,575	·01690	18·746	97	46	20	·43478	1·040
53	151,197	2,809	·01858	18·068	98	25	13	·52000	·840
54	149,048	2,829	·01898	17·410	99	8	5	·62500	·750
55	147,015	3,087	·02100	16·747	100	3	2	·66667	1·000
56	144,099	3,253	·02257	16·106	101	1	0	·00000	2·000
57	140,493	3,369	·02398	15·478	102	1	0	·00000	1·000
58	136,620	3,260	·02386	14·858	103	1	1	1·00000	·000
59	132,824	3,552	·02674	14·221					
60	128,559	3,797	·02954	13·612					
61	123,571	3,671	·02971	13·026					
62	118,496	3,878	·03273	12·425					
63	112,891	4,038	·03577	11·846					
64	107,244	4,244	·03957	11·285					

(44). The first step was the graduation of the Select Table (combined ages at entry) excluding the first ten years from entry, and it will be convenient here to give in full (*see* Table XXII) the table of exposed to risk and died, together with the unadjusted rates of mortality and curtate expectations of life, as this table has not been elsewhere published, although the data from which it has been constructed are of course to be found in the volume of the unadjusted mortality experience.

It was obviously desirable that the graduation of this table should proceed as far as possible on similar lines to the $O^{M(5)}$ Table. The graduation was therefore made by Makeham's formula, the value of log c being assumed equal to ·039 as in the former table, and the constants A and B were determined in the method already described, viz., by equating to zero the total of the deviations and of the accumulated deviations between the adjusted and actual deaths. Table XXIII will show the extent of agreement between the graduated and ungraduated tables.

TABLE XXIII.

Whole-Life Participating Assurance Experience—Males.

Truncated Table, **TEN** *Years (Select Data).*

Log c = ·039 α = ·0026111 β = ·000046635 in $\text{colog}_{10}(p_x) = \alpha + \beta c^x$

Group of Ages	Actual Deaths	Expected Deaths	Expected Deviation	Actual Deviation		Accumulated Deviation	
			±	+	−	+	−
20–4	15	12	3	...	3	2	...
25–9	113	116	8	3	...	5	...
30–4	864	862	23	...	2	2	...
35–9	3,119	3,136	43	17	...	4	...
40–4	6,461	6,300	62	...	161	...	13
45–9	9,761	9,698	77	...	63	148	...
50–4	13,071	13,183	88	112	...	211	...
55–9	16,521	16,636	99	115	...	99	...
60–4	19,628	19,816	108	188	...	...	16
65–9	21,428	21,527	113	99	...	...	204
70–4	20,731	20,505	112	...	226	...	303
75–9	16,160	16,105	98	...	55	...	77
80–4	10,003	9,796	77	...	207	...	22
85–9	3,946	4,131	49	185	...	185	...
Total ...	141,821	141,823	±960	719	717	656	635
				±1,436		±1,291	

(45). The deviations at individual groups of ages between the actual and expected deaths may appear at first sight rather large, and

are on the average about 50 per cent. in excess of the expected deviations (these latter, however, being under-estimated, owing to the inclusion of duplicate assurances upon the same life). In other words, they are not entirely due to accidental errors of observations, but in part to the fact that Makeham's curve does not exactly fit the table. They are, however, on the whole fairly well balanced, and the general features of the ungraduated tables are well reproduced, as will be seen from the following comparison of the actual and expected deaths in larger age groups :—

TABLE XXIV.

Whole-Life Participating Assurance Experience—Males.

Truncated Table,* TEN *Years (Select Data).

Comparison of Deaths in larger age-groups.

Group of Ages	Actual Deaths	Graduated Deaths	Deviation (Graduated - Actual)
20–39	4,111	4,126	+15
40–59	45,814	45,817	+ 3
60–79	77,947	77,953	+ 6
80–89	13,949	13,927	−22

(46). The general agreement between the ungraduated and graduated tables is also well shown in the following Table XXV of the values of the expectations of life, from which it will be seen that the average deviation irrespective of sign is only ±·017.

TABLE XXV.

Whole-Life Participating Assurance Experience—Males.

Truncated Table,* TEN *Years (Select Data).

Comparison of Graduated and Ungraduated Expectations.

Ages at Entry	Ages Attained	Mean of Five Values of e_x at Consecutive Ages		Deviation (G−U)	
		Ungraduated	Graduated	+	−
15–19	25–29	36·76	36·76	...	...
20–24	30–34	33·06	33·08	·02	...
25–29	35–39	29·40	29·40	...	...
30–34	40–44	25·74	25·75	·01	...
35–39	45–49	22·20	22·17	...	·03
40–44	50–54	18·75	18·72	...	·03
45–49	55–59	15·48	15·44	...	·04
50–54	60–64	12·44	12·44	...	...
55–59	65–69	9·71	9·73	·02	...
60–64	70–74	7·35	7·37	·02	...
Totals		210·89	210·86	·07	·10
Average per age ...		21·089	21·086	...	·003

(47). The adjustment of the rates of mortality for the first 10 years after entry was made in the following manner:—

A graduation was made for each year of assurance, 0 to 9, in the same manner as for the aggregate table, and a series of values thus obtained for the constants α and β, it being assumed that the force of mortality in the middle of the year $= \frac{1}{M}\text{colog}_{10}(p_{[x]+t})$. The progress of these values from year to year is necessarily very irregular, owing to the somewhat limited data for individual insurance years. They can, therefore, only be used as a general indication of the nature of the variation in the two constants from year to year.

(48). Speaking generally, it may be said that the effect of selection upon the constant α is much the greater for the first two or three years after entry, but is somewhat rapidly exhausted, not being very important after the fifth year. The effect on the constant β is, however, much more durable, and has by no means worn off at the end of the ten years. This is illustrated in the following Table XXVI, giving approximate values of these constants for each of the first 10 years of assurance, and for the ultimate table.

TABLE XXVI.

Whole-Life Participating Assurance Experience—Males

Select Experience first **TEN** *Years.*

Approximate Values of Constants in formula $\text{colog}_{10}(p_x) = \alpha + \beta c^x$.

Year of Assurance	α	β
0	·00098	·0000259
1	·00152	·0000354
2	·00188	·0000353
3	·00193	·0000396
4	·00211	·0000381
5	·00222	·0000428
6	·00267	·0000375
7	·00244	·0000443
8	·00264	·0000407
9	·00255	·0000447
10 and upwards	·00261	·0000466

(49). In determining the form of curve during the first ten years, account had to be taken of the same necessary conditions as in the case of the annuity experience, that is to say, while adhering as closely as possible to the facts for various age groups and durations of assurance, it was necessary to effect a smooth juncture between the select and aggregate curves, and it was very

desirable to adopt expressions for the difference between the select and aggregate mortality during the first ten years, such as would permit of the exact calculation of the values of the force of mortality for each year.

(50). As the variations in the constant β were on the whole the more intractable, this constant was dealt with in the first instance, and owing to the persistence of the effect of selection on its value, it was necessary to adopt a curve for the values of $l_{[x]+t}$ which, while tangential to the ultimate curve for the value $t=10$, should leave it sufficiently rapidly to follow approximately the unadjusted curve. To this end it was assumed that the differences between the two curves $\log l_{x+t}$ and $\log l_{[x]+t}$, so far as this difference depended on a change in the β constant, was represented by a parabolic curve, as in equations (21) and (23) below. It was then assumed that the deviations of the constant a from its ultimate value would form a similar parabolic curve near the point of juncture between the select and aggregate tables, together with a rapidly diminishing geometrical series, representing the rapid rise in the value of this constant in the first year or two after entry.

(51). The formula for $\log l_{[x]+t}$ for values of t from 0 to 10 thus became—

$$\log_{10} l_{[x]+t} = \log_{10} l_{x+t} - f_t - \beta c^x \psi_t \quad . \quad . \quad . \quad . \quad (21)$$

where
$$f_t = m(10-t)^2 + m'(c')^t \quad . \quad . \quad . \quad . \quad (22)$$

and
$$\psi_t = n(10-t)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

The values ultimately adopted for m, m', and n, were respectively ·000040955, ·00112, and ·02386. As the value of c' worked out at ·24, the second term of f_t rapidly became insignificant.

(52). The general form of curve having thus been decided upon, the numerical values of the constants, there being four unknown quantities, were determined by equating to zero :—

(1) The total of the deviations between the actual and expected deaths in the first ten years of assurance, for all ages at entry from 15 to 74.

(2) The total of the accumulated deviations, when arranged in quinquennial groups of ages.

(3) The total deviations for the above ages at entry, for the year of assurance 0.

(4) The total of the accumulated deviations, taken for quinquennial groups of ages for year of assurance 0.

As a matter of fact, the values of the constants so determined were also found to give a very close agreement between the totals of

TABLE XXVII.

Whole-Life Participating Assurance Experience—Males.

First **TEN** *Years of Assurance (Select Data).*

Expected Deaths (Ordinary Type). Log $c = .039$. **Actual Deaths (Black type).**

Grouped Ages at Date of Assurance	Years Elapsed Since Date of Assurance										Total for each Group of Ages at Date of Assurance
	0	1	2	3	4	5	6	7	8	9	
15–9	25·3	38·6	41·1	41·7	42·1	41·9	41·7	41·6	41·7	41·7	397·4
	28	**46**	**35**	**43**	**50**	**52**	**39**	**37**	**45**	**52**	**427**
20–4	165·2	247·3	262·2	266·2	269·1	271·5	273·4	275·6	277·8	280·4	2588·7
	158	**261**	**278**	**267**	**268**	**274**	**305**	**301**	**283**	**294**	**2689**
25–9	318·5	476·6	516·6	533·7	547·9	560·4	571·0	583·2	593·7	605·5	5307·1
	305	**417**	**485**	**511**	**528**	**531**	**628**	**592**	**573**	**584**	**5154**
30–4	339·2	500·3	546·5	573·0	595·3	616·4	634·7	655·8	676·7	699·5	5837·4
	366	**466**	**546**	**550**	**592**	**682**	**669**	**626**	**713**	**705**	**5915**
35–9	297·2	426·6	470·7	499·5	525·8	551·8	577·3	606·3	632·6	661·9	5249·7
	277	**433**	**479**	**505**	**519**	**534**	**584**	**627**	**599**	**588**	**5145**
40–4	252·2	349·7	390·3	420·4	448·5	476·7	503·5	534·0	561·8	591·9	4529·0
	272	**341**	**392**	**426**	**409**	**459**	**486**	**559**	**571**	**558**	**4473**
45–9	204·5	274·7	308·4	337·2	362·2	387·5	412·0	440·8	466·1	495·9	3689·3
	190	**290**	**302**	**373**	**369**	**417**	**389**	**467**	**445**	**497**	**3739**
50–4	158·5	206·2	232·3	254·5	278·0	300·7	320·9	344·2	367·0	391·5	2853·8
	168	**246**	**261**	**267**	**275**	**340**	**345**	**305**	**335**	**406**	**2948**
55–9	107·8	136·1	154·4	172·2	189·9	205·7	221·7	240·4	255·8	272·5	1956·5
	109	**155**	**167**	**172**	**175**	**181**	**198**	**245**	**242**	**250**	**1894**
60–4	64·8	80·9	91·5	101·6	111·0	119·7	127·3	134·1	140·3	147·8	1119·0
	64	**88**	**95**	**91**	**126**	**143**	**117**	**139**	**116**	**153**	**1132**
65–9	30·6	36·8	42·0	46·6	50·2	52·6	54·7	57·7	57·9	59·9	489·0
	26	**45**	**39**	**59**	**59**	**55**	**48**	**72**	**48**	**63**	**514**
70–4	10·6	12·8	15·2	16·8	17·5	18·3	18·2	18·7	19·2	18·5	165·8
	11	**10**	**10**	**22**	**21**	**14**	**16**	**16**	**13**	**20**	**153**
Total for each year of Assurance	1974·4	2786·6	3071·2	3263·4	3437·5	3603·2	3756·4	3932·4	4090·6	4267·0	34182·7
	1974	**2798**	**3089**	**3286**	**3391**	**3682**	**3824**	**3986**	**3983**	**4170**	**34183**

Table XXVIII.

Whole-Life Participating Assurance Experience.—Males.

First **TEN** *Years of Assurance.* (*Select Data.*)

(+ Errors in Ordinary Type). Expected Deaths, less Actual Deaths. (− Errors in Black Type).

Grouped Ages at Date of Assurance	Years Elapsed Since Date of Assurance										Total for each Group of Ages at Date of Assurance
	0	1	2	3	4	5	6	7	8	9	
15–9	− 2·7	− 7·4	+ 6·1	− 1·3	− 7·9	−10·1	+ 2·7	+ 4·6	− 3·3	−10·3	− 29·6
20–4	+ 7·2	−13·7	−15·8	− ·8	+ 1·1	− 2·5	−31·6	−25·4	− 5·2	−13·6	−100·3
25–9	+13·5	+59·6	+31·6	+22·7	+20·1	+29·4	−57·0	− 8·8	+ 20·7	+21·5	+153·1
30–4	−26·8	+34·3	+ ·5	+23·0	+ 3·3	−65·6	−34·3	+29·8	− 36·3	− 5·5	− 77·6
35–9	+20·2	− 6·4	− 8·3	− 5·5	+ 6·8	+17·8	− 6·7	−20·7	+ 33·6	+73·9	+104·7
40–4	−19·8	+ 8·7	− 1·7	− 5·6	+39·5	+17·7	+17·5	−25·0	− 9·2	+33·9	+ 56·0
45–9	+14·5	−15·3	+ 6·4	−35·8	− 6·8	−29·5	+23·0	−26·2	+ 21·1	− 1·1	− 49·7
50–4	− 9·5	−39·8	−28·7	−12·5	+ 3·0	−39·3	−24·1	+39·2	+ 32·0	−14·5	− 94·2
55–9	− 1·2	−18·9	−12·6	+ ·2	+14·9	+24·7	+23·7	− 4·6	+ 13·8	+22·5	+ 62·5
60–4	+ ·8	− 7·1	− 3·5	+10·6	−15·0	−23·3	+10·3	− 4·9	+ 24·3	− 5·2	− 13·0
65–9	+ 4·6	− 8·2	+ 3·0	−12·4	− 8·8	− 2·4	+ 6·7	−14·3	+ 9·9	− 3·1	− 25·0
70–4	− ·4	+ 2·8	+ 5·2	+ 5·2	− 3·5	+ 4·3	+ 2·2	+ 2·7	+ 6·2	− 1·5	+ 12·8
Total ...	+ ·4	−11·4	−17·8	−22·6	+46·5	−78·8	−67·6	−53·6	+107·6	+97·0	− ·3

the expected and actual deaths for all ages for years 1 to 4, and for years 5 to 9, of assurance.

(53). The expected deaths in all cases were computed from the value of $\mu_{x+\frac{1}{2}}$ combined with the exposed to risk in the middle of each year. They were subsequently, however, computed by the values of q_x in the finally adjusted Table XXVII, where the expected deaths are taken to one decimal place, the integers representing the actual deaths. For greater clearness, the deviations are shown separately in Table XXVIII.

(54). On the whole, it will be seen that changes of sign in the deviations are frequent, and that there is a close agreement in the general characteristics of the unadjusted and adjusted tables. This general agreement is also apparent on a comparison of the graduated and ungraduated expectations at the date of entry, as shown in Table XXIX, the slight defect of the graduated tables as compared with the ungraduated on the average, being due to the fact that selection, as already stated, has not entirely worn off at the end of ten years from the date of assurance, and to exclusion of the data above age 90.

TABLE XXIX.

Whole-Life Participating Assurance Experience—Males. Select Tables.

Comparison of Expectations at entry, by *(a)* Graduated Table joining after 10 Years on to Truncated 10-Year Table (Select Data), with *(b)* Ungraduated Expectations by Select Table. (One Age at Entry throughout.)

Ages at Entry	Mean of 5 Values of Expectations at Entry		Deviation (G − U)	
	Ungraduated (*b*)	Graduated (*a*)	+	−
15–19	44·35	44·54	·19	...
20–24	40·91	40·93	·02	...
25–29	37·57	37·28	...	·29
30–34	33·56	33·60	·04	...
35–39	29·95	29·92	...	·03
40–44	26·36	26·29	...	·07
45–49	22·62	22·76	·14	...
50–54	19·38	19·37	...	·01
55–59	16·28	16·18	...	·10
60–64	13·38	13·26	...	·12
Totals ...	284·36	284·13	·39	·62
Average per Age ...	28·436	28·413	...	·023

(55). Similar comparisons between the graduated and ungraduated values of the annual whole-life premium at the date of entry is given in the following Table XXX, where the ungraduated values have been estimated by means of relative expectations of life.

TABLE XXX.

Whole-Life Participating Assurance Experience—Males Select Tables.

Comparison of Graduated Annual Premiums at date of assurance with estimated Ungraduated Premiums, and with Sprague's Select Premiums (H^M Data). 3 per cent. Interest.

Age	Ungraduated $P_{[x]}$	Graduated $P_{[x]}$	Deviation G−U		Sprague's Select $P_{[x]}$	Sprague —New
			+	−		+
20	1·379	1·365	...	·014	1·563	·198
25	1·535	1·551	·016	...	1·703	·151
30	1·779	1·785	·006	...	1·925	·140
35	2·086	2·081	...	·005	2·218	·137
40	2·453	2·457	·004	...	2·602	·145
45	2·952	2·940	...	·012	3·106	·166
50	3·571	3·564	...	·007	3·755	·191
55	4·338	4·377	·039	...	4·635	·258
60	5·413	5·446	·033	...	5·827	·381
65	6·872	6·854	...	·018	7·433	·579
Average	3·238	3·242	·004	...	3·477	·235

It will be seen that in no case does the deviation exceed one per cent. of the premium, the average graduated premiums for all ages at entry, from 20 to 65, being in excess of the ungraduated by exactly 1*d.* per cent.

(56). In the following Table XXXI is given a complete statement of the values of the various functions employed, and Table XXXII shows the values of colog $p_{[x]+t}$ for each of the first ten years of assurance, and of colog p_x for the ultimate table.

(57). It may be noted, in conclusion, that the values of the select functions for the Annuitant Experience have been carried to age 99, as they may be occasionally useful at advanced ages. In the case of the Assurance Experience, they cease at age 75, as in the absence of data it would be unsafe to publish them for older ages.

G. F. HARDY.

Table XXXI.

Whole-Life Participating Assurance Experience—Males.

Select Tables—$0^{[24]}$

Graduation Functions.

t	A_t	B_t	a_t	β_t	$\log_{10} B_t$	$\log_{10} \beta_t$	$\log_{10} k_t$	$\log_{10} g_t$
0	·000 445 9	·000 051 393	·000 981 8	·000 025 494	$\bar{5}$·7 109 040	$\bar{5}$·4 064 380	5·027 602 5	—·000 607 615
1	·003 431 5	·000 060 475	·001 710 6	·000 029 345	$\bar{5}$·7 815 758	$\bar{5}$·4 675 341	5·029 231 8	—·000 578 733
2	·004 291 4	·000 068 378	·001 947 8	·000 032 689	$\bar{5}$·8 349 179	$\bar{5}$·5 144 016	5·030 132 3	—·000 555 852
3	·004 641 2	·000 075 236	·002 066 9	·000 035 586	$\bar{5}$·8 764 230	$\bar{5}$·5 512 792	5·030 795 6	—·000 537 993
4	·004 868 4	·000 081 165	·002 157 8	·000 038 089	$\bar{5}$·9 093 705	$\bar{5}$·5 807 996	5·031 339 8	—·000 524 316
5	·005 066 3	·000 086 279	·002 241 8	·000 040 243	$\bar{5}$·9 359 059	$\bar{5}$·6 046 903	5·031 793 1	—·000 514 102
6	·005 257 2	·000 090 673	·002 324 2	·000 042 091	$\bar{5}$·9 574 790	$\bar{5}$·6 241 892	5·032 162 4	—·000 506 734
7	·005 446 3	·000 094 433	·002 406 3	·000 043 668	$\bar{5}$·9 751 254	$\bar{5}$·6 401 633	5·032 449 3	—·000 501 688
8	·005 635 0	·000 097 635	·002 488 2	·000 045 008	$\bar{5}$·9 896 072	$\bar{5}$·6 532 897	5·032 654 1	—·000 498 517
9	·005 823 7	·000 100 348	·002 570 2	·000 046 139	$\bar{4}$·0 015 086	$\bar{5}$·6 640 682	5·032 777 0	—·000 496 843
10 or more	·006 012 3	·000 102 632	·002 611 1	·000 046 635	$\bar{4}$·0 112 814	$\bar{5}$·6 687 120	5·032 817 9	—·000 496 347

Absolute Constants.

$c = 1{\cdot}093\,956\,4$; $\log_{10} c = {\cdot}039$; $\log_e c = {\cdot}089\,800\,8$; $\log_{10} s = -a_{10} = -{\cdot}002\,611\,1$.

TABLE XXXII.

BRITISH OFFICES LIFE TABLES, 1893.

Whole-Life Participating Assurances—Males.

Select Tables—$0^{[16]}$.

Values of $\text{colog}_{10}(p_{[x]+t})$ and of $\text{colog}_{10}(p_{x+10})$.

Age at Entry $[x]$	0	1	2	3	4	5	6	7	8	9	10 or more	Age attained $x+10$
	$\text{col}_{10}(p_{[x]+0})$	$\text{col}_{10}(p_{[x]+1})$	$\text{col}_{10}(p_{[x]+2})$	$\text{col}_{10}(p_{[x]+3})$	$\text{col}_{10}(p_{[x]+4})$	$\text{col}_{10}(p_{[x]+5})$	$\text{col}_{10}(p_{[x]+6})$	$\text{col}_{10}(p_{[x]+7})$	$\text{col}_{10}(p_{[x]+8})$	$\text{col}_{10}(p_{[x]+9})$	$\text{col}_{10}(p_{x+10})$	
10	·001 044 4	·001 789 4	·002 043 8	·002 181 2	·002 291 7	·002 396 6	·002 501 3	·002 607 3	·002 714 8	·002 824 3	·002 892 1	20
11	·001 050 3	·001 796 8	·002 052 9	·002 192 0	·002 304 3	·002 411 1	·002 517 9	·002 626 2	·002 736 1	·002 848 1	·002 918 5	21
12	·001 056 7	·001 804 9	·002 062 7	·002 203 8	·002 318 0	·002 427 0	·002 536 1	·002 646 8	·002 759 4	·002 874 2	·002 947 4	22
13	·001 063 7	·001 813 8	·002 073 5	·002 216 6	·002 333 1	·002 444 4	·002 556 1	·002 669 4	·002 784 9	·002 902 8	·002 979 0	23
14	·001 071 4	·001 823 4	·002 085 4	·002 230 7	·002 349 6	·002 463 5	·002 577 8	·002 694 2	·002 812 8	·002 934 1	·003 013 5	24
15	·001 079 8	·001 834 1	·002 098 3	·002 246 1	·002 367 6	·002 484 3	·002 601 7	·002 721 2	·002 843 3	·002 968 3	·003 051 4	25
16	·001 089 1	·001 845 6	·002 112 4	·002 262 9	·002 387 3	·002 507 1	·002 627 7	·002 750 8	·002 876 6	·003 005 7	·003 092 7	26
17	·001 099 1	·001 858 4	·002 127 8	·002 281 3	·002 408 9	·002 532 0	·002 656 2	·002 783 1	·002 913 1	·003 046 6	·003 138 0	27
18	·001 110 2	·001 872 2	·002 144 8	·002 301 5	·002 432 4	·002 559 2	·002 687 4	·002 818 6	·002 953 0	·003 091 4	·003 187 5	28
19	·001 122 2	·001 887 4	·002 163 3	·002 323 5	·002 458 3	·002 589 1	·002 721 6	·002 857 3	·002 996 7	·003 140 3	·003 241 6	29
20	·001 135 4	·001 904 0	·002 183 5	·002 347 6	·002 486 5	·002 621 7	·002 758 9	·002 899 7	·003 044 5	·003 193 9	·003 300 9	30
21	·001 149 9	·001 922 2	·002 205 7	·002 374 0	·002 517 4	·002 657 4	·002 799 7	·002 946 0	·003 096 7	·003 252 6	·003 365 7	31
22	·001 165 6	·001 942 1	·002 229 9	·002 402 9	·002 551 2	·002 696 5	·002 844 4	·002 996 7	·003 153 9	·003 316 7	·003 436 6	32
23	·001 182 9	·001 963 9	·002 256 4	·002 434 4	·002 588 1	·002 739 2	·002 893 3	·003 052 2	·003 216 5	·003 386 8	·003 514 1	33
24	·001 201 8	·001 987 6	·002 285 4	·002 468 9	·002 628 6	·002 785 9	·002 946 8	·003 112 9	·003 284 9	·003 463 5	·003 599 0	34
25	·001 222 5	·002 013 7	·002 317 1	·002 506 7	·002 672 8	·002 837 0	·003 005 3	·003 179 3	·003 359 8	·003 547 5	·003 691 8	35
26	·001 245 1	·002 042 1	·002 351 8	·002 548 0	·002 721 2	·002 893 0	·003 069 3	·003 251 9	·003 441 6	·003 639 3	·003 793 4	36
27	·001 269 8	·002 073 3	·002 389 8	·002 593 3	·002 774 1	·002 954 1	·003 139 3	·003 331 3	·003 531 2	·003 739 8	·003 904 4	37
28	·001 296 9	·002 107 4	·002 431 3	·002 642 7	·002 832 0	·003 021 1	·003 215 8	·003 418 3	·003 629 2	·003 849 7	·004 026 0	38
29	·001 326 5	·002 144 6	·002 476 8	·002 696 8	·002 895 4	·003 094 3	·003 299 6	·003 513 3	·003 736 4	·003 969 9	·004 158 9	39
30	·001 358 9	·002 185 4	·002 526 4	·002 756 0	·002 964 7	·003 174 4	·003 391 3	·003 617 4	·003 853 7	·004 101 4	·004 304 3	40
31	·001 394 3	·002 230 0	·002 580 8	·002 820 7	·003 040 5	·003 262 0	·003 491 5	·003 731 1	·003 982 0	·004 245 3	·004 463 4	41
32	·001 433 1	·002 278 9	·002 640 3	·002 891 6	·003 123 4	·003 357 9	·003 601 2	·003 855 6	·004 122 3	·004 402 7	·004 637 5	42
33	·001 475 5	·002 332 2	·002 705 3	·002 969 1	·003 214 1	·003 462 7	·003 721 2	·003 991 8	·004 275 9	·004 574 9	·004 827 8	43
34	·001 521 8	·002 390 7	·002 776 5	·003 053 8	·003 313 4	·003 577 4	·003 852 4	·004 140 8	·004 443 8	·004 763 3	·005 036 1	44
35	·001 572 6	·002 454 5	·002 854 4	·003 146 5	·003 422 0	·003 702 9	·003 996 0	·004 303 7	·004 627 6	·004 969 3	·005 263 9	45
36	·001 628 1	·002 524 4	·002 939 5	·003 248 0	·003 540 7	·003 840 2	·004 153 1	·004 482 0	·004 828 6	·005 194 7	·005 513 2	46
37	·001 688 8	·002 600 9	·003 032 7	·003 359 0	·003 670 7	·003 990 4	·004 324 9	·004 677 0	·005 048 5	·005 441 4	·005 785 9	47
38	·001 755 3	·002 684 5	·003 134 7	·003 480 3	·003 812 8	·004 154 7	·004 512 9	·004 890 4	·005 289 0	·005 711 1	·006 084 2	48
39	·001 827 9	·002 776 1	·003 246 2	·003 613 2	·003 968 3	·004 334 4	·004 718 6	·005 123 8	·005 552 2	·006 006 2	·006 410 5	49
40	·001 907 4	·002 876 1	·003 368 2	·003 758 4	·004 138 4	·004 531 0	·004 943 5	·005 379 1	·005 840 1	·006 329 1	·006 767 4	50
41	·001 994 4	·002 985 7	·003 501 6	·003 917 4	·004 324 5	·004 746 1	·005 189 6	·005 658 4	·006 155 0	·006 682 2	·007 158 0	51
42	·002 089 6	·003 105 5	·003 647 6	·004 091 2	·004 528 1	·004 981 4	·005 458 9	·005 963 9	·006 499 6	·007 068 6	·007 585 2	52
43	·002 193 6	·003 236 5	·003 807 3	·004 281 4	·004 750 8	·005 238 8	·005 753 4	·006 298 2	·006 876 4	·007 491 3	·008 052 5	53
44	·002 307 5	·003 379 9	·003 982 1	·004 489 5	·004 994 4	·005 520 4	·006 075 6	·006 663 9	·007 288 7	·007 953 6	·008 563 8	54

45	·002 432 0	·003 536 7	·004 173 2	·004 717 1	·005 260 9	·005 828 5	·006 428 0	·007 063 9	·007 739 8	·008 459 5	·009 123 1	55
46	·002 568 3	·003 708 3	·004 382 3	·004 966 1	·005 552 5	·006 165 4	·006 813 6	·007 501 5	·008 233 2	·009 012 8	·009 734 9	56
47	·002 717 4	·003 896 0	·004 611 0	·005 238 5	·005 871 4	·006 534 1	·007 235 4	·007 980 2	·008 773 0	·009 618 1	·010 404 2	57
48	·002 880 4	·004 101 4	·004 861 2	·005 536 5	·006 220 4	·006 937 4	·007 696 9	·008 504 0	·009 363 5	·010 280 3	·011 136 4	58
49	·003 058 8	·004 326 0	·005 134 9	·005 862 5	·006 602 1	·007 378 6	·008 201 7	·009 076 9	·010 009 4	·011 004 8	·011 937 5	59
50	·003 253 9	·004 571 7	·005 434 4	·006 219 1	·007 019 6	·007 861 2	·008 753 9	·009 703 6	·010 716 1	·011 797 3	·012 813 7	60
51	·003 467 4	·004 840 5	·005 762 0	·006 609 2	·007 476 4	·008 389 2	·009 358 0	·010 389 2	·011 489 2	·012 664 2	·013 772 3	61
52	·003 701 0	·005 134 6	·006 120 3	·007 036 0	·007 976 2	·008 966 8	·010 018 9	·010 139 3	·012 334 9	·013 612 6	·014 821 0	62
53	·003 956 5	·005 456 3	·006 512 4	·007 502 9	·008 522 8	·009 598 6	·010 741 8	·011 959 8	·013 260 0	·014 650 1	·015 968 2	63
54	·004 236 0	·005 808 3	·006 941 3	·008 013 7	·009 120 9	·010 289 8	·011 532 7	·012 857 4	·014 272 1	·015 785 1	·017 223 2	64
55	·004 541 7	·006 193 2	·007 410 4	·008 572 4	·009 775 1	·011 046 0	·012 397 9	·013 839 4	·015 379 3	·017 026 8	·018 596 1	65
56	·004 876 2	·006 614 4	·007 923 7	·009 183 6	·010 490 7	·011 873 2	·013 344 4	·014 913 6	·016 590 5	·018 385 1	·020 098 0	66
57	·005 242 1	·007 075 2	·008 485 1	·009 852 3	·011 273 7	·012 778 1	·014 379 8	·016 088 6	·017 915 5	·019 871 0	·021 741 0	67
58	·005 642 4	·007 579 2	·009 099 4	·010 583 7	·012 130 2	·013 768 1	·015 512 5	·017 374 3	·019 365 0	·021 496 5	·023 538 3	68
59	·006 080 2	·008 130 6	·009 771 3	·011 384 0	·013 067 2	·014 851 1	·016 751 6	·018 780 6	·020 950 7	·023 274 8	·025 504 6	69
60	·006 559 3	·008 733 8	·010 506 4	·012 259 4	·014 092 2	·016 035 8	·018 107 2	·020 319 1	·022 685 3	·025 220 1	·027 655 6	70
61	·007 083 3	·009 393 7	·011 310 5	·013 217 0	·015 213 5	·017 331 8	·019 590 1	·022 002 1	·024 583 0	·027 348 2	·030 008 7	71
62	·007 656 6	·010 115 5	·012 190 2	·014 264 6	·016 440 1	·018 749 6	·021 212 3	·023 843 3	·026 658 9	·029 676 3	·032 582 7	72
63	·008 283 7	·010 905 2	·013 152 5	·015 410 7	·017 782 1	·020 300 7	·022 987 0	·025 857 4	·028 929 9	·032 223 1	·035 398 9	73
64	·008 969 8	·011 769 1	·014 205 3	·016 664 4	·019 250 0	·021 997 4	·024 928 4	·028 060 8	·031 414 3	·035 009 2	·038 479 5	74
65	·009 720 3	·012 714 2	·015 357 1	·018 036 0	·020 856 0	·023 853 6	·027 052 2	·030 471 2	·034 132 1	·038 057 0	·041 849 6	75
66	·010 541 4	·013 748 0	·016 616 8	·019 536 3	·022 612 8	·025 884 1	·029 375 6	·033 108 1	·037 105 2	·041 391 2	·045 536 3	76
67	·011 439 5	·014 879 0	·017 995 1	·021 177 7	·024 534 7	·028 105 5	·031 917 2	·035 992 7	·040 357 7	·045 038 7	·049 569 4	77
68	·012 422 1	·016 116 3	·019 502 8	·022 973 3	·026 637 1	·030 535 5	·034 697 7	·039 148 3	·043 915 8	·049 028 9	·053 981 4	78
69	·013 497 0	·017 469 8	·021 152 2	·024 937 6	·028 937 1	·033 193 9	·037 739 4	·042 600 5	·047 808 2	·053 394 0	·058 808 0	79
70	·014 672 9	·018 950 5	·022 956 6	·027 086 4	·031 453 2	·036 102 1	·041 066 8	·046 377 0	·052 066 3	·058 169 2	·064 088 0	80
71	·015 959 3	·020 570 3	·024 930 5	·029 437 2	·034 205 7	·039 283 4	·044 707 0	·050 508 3	·056 724 5	·063 393 1	·069 864 1	81
72	·017 366 5	·022 342 3	·027 089 9	·032 008 8	·037 216 8	·042 763 7	·048 689 1	·055 027 8	·061 820 3	·069 107 9	·076 183 0	82
73	·018 905 9	·024 280 7	·029 452 2	·034 822 0	·040 510 8	·046 571 1	·053 045 4	·059 971 9	·067 394 9	·075 359 5	·083 095 5	83
74	·020 590 0	·026 401 3	·032 036 4	·037 899 6	·044 114 4	·050 736 1	·057 810 9	·065 380 6	·073 493 3	·082 198 5	·090 657 6	84
75	·022 432 3	·028 721 2	·034 863 4	·041 266 3	·048 056 4	·055 292 4	·063 024 3	·071 297 4	·080 164 7	·089 680 1	·098 930 1	85
											·107 979 9	86
											·117 880 0	87
											·128 710 2	88
											·140 558 0	89
											·153 519 0	90
											·167 697 8	91
											·183 208 8	92
											·200 177 1	93
											·218 739 6	94
											·239 046 2	95
											·261 260 8	96
											·285 562 6	97
											·312 147 7	98
											·341 230 6	99
											·373 046 1	100
											·407 850 8	101
											·445 925 7	102
											·487 577 9	103
											·533 143 7	104

TABLE XIA.

Female Annuitant Experience.

Values of $r_{[x]+t} = l^{(2)}_{[x]+t} \div l^{(1)}_{[x]+t}$.

Age at Entry [x]	Years elapsed since Date of Purchase (t)						Age attained $x+5$
	0	1	2	3	4	5 or more	
20	·74679	·72753	·70776	·68745	·66669	·64573	25
21	·72439	·70549	·68607	·66606	·64558	·62489	26
22	·70231	·68377	·66467	·64495	·62474	·60428	27
23	·68053	·66234	·64354	·62410	·60413	·58390	28
24	·65904	·64118	·62268	·60349	·58375	·56373	29
25	·63782	·62028	·60205	·58310	·56358	·54376	30
26	·61684	·59961	·58164	·56293	·54360	·52396	31
27	·59609	·57916	·56144	·54294	·52380	·50432	32
28	·57556	·55891	·54144	·52313	·50416	·48484	33
29	·55523	·53885	·52160	·50349	·48468	·46550	34
30	·53508	·51896	·50193	·48399	·46534	·44629	35
31	·51510	·49924	·48241	·46464	·44612	·42720	36
32	·49528	·47966	·46303	·44542	·42703	·40823	37
33	·47560	·46022	·44378	·42632	·40806	·38938	38
34	·45605	·44090	·42465	·40734	·38921	·37064	39
35	·43663	·42171	·40564	·38847	·37047	·35202	40
36	·41733	·40264	·38674	·36972	·35185	·33353	41
37	·39815	·38368	·36796	·35109	·33335	·31517	42
38	·37909	·36484	·34930	·33259	·31499	·29697	43
39	·36015	·34612	·33077	·31422	·29679	·27893	44
40	·34133	·32753	·31238	·29601	·27875	·26110	45
41	·32265	·30909	·29415	·27797	·26091	·24348	46
42	·30412	·29081	·27609	·26013	·24330	·22613	47
43	·28577	·27272	·25824	·24251	·22595	·20907	48
44	·26761	·25483	·24061	·22516	·20889	·19236	49
45	·24967	·23719	·22326	·20811	·19219	·17605	50
46	·23200	·21982	·20621	·19141	·17588	·16019	51
47	·21462	·20278	·18952	·17511	·16001	·14483	52
48	·19759	·18612	·17324	·15926	·14466	·13004	53
49	·18095	·16987	·15743	·14392	·12988	·11588	54
50	·16476	·15411	·14213	·12916	·11572	·10242	55
51	·14909	·13889	·12742	·11504	·10227	·08970	56
52	·13398	·12427	·11335	·10161	·08956	·07780	57
53	·11950	·11031	·10000	·08894	·07767	·06676	58
54	·10573	·09709	·08741	·07708	·06663	·05661	59
55	·09271	·08465	·07564	·06609	·05650	·04740	60
56	·08050	·07305	·06475	·05600	·04730	·03914	61
57	·06917	·06234	·05477	·04684	·03905	·03183	62
58	·05875	·05256	·04572	·03864	·03175	·02546	63
59	·04927	·04372	·03764	·03139	·02539	·02000	64
60	·04075	·03584	·03050	·02507	·01993	·01540	65
61	·03320	·02893	·02431	·01967	·01535	·01163	66
62	·02661	·02294	·01901	·01512	·01156	·00854	67
63	·02095	·01785	·01458	·01138	·00851	·00613	68
64	·01617	·01361	·01093	·00836	·00610	·00427	69
65	·01222	·01015	·00800	·00599	·00425	·00289	70
66	·00902	·00738	·00571	·00417	·00288	·00189	71
67	·00649	·00522	·00396	·00281	·00188	·00119	72
68	·00454	·00359	·00266	·00183	·00119	·00072	73
69	·00309	·00239	·00173	·00115	·00072	·00042	74
70	·00203	·00154	·00108	·00070	·00042	·00023	75
71	·00128	·00095	·00065	·00040	·00023	·00012	76
72	·00078	·00057	·00037	·00022	·00012	·00006	77
73	·00046	·00032	·00020	·00012	·00006	·00003	78
74	·00025	·00017	·00011	·00006	·00003	·00001	79
75	·00013	·00009	·00005	·00003	·00001	...	80
76	·00007	·00004	·00002	·00001	...	...	81
77	·00003	·00002	·00001	...	...	...	82
78	·00001	·00001	...	...	...	...	83
79	·00001	...	...	...	...	...	84

NOTES ON THE METHODS ADOPTED

IN THE

CONSTRUCTION OF MORTALITY AND MONETARY TABLES,

DERIVED FROM THE

EXPERIENCE OF LIVES ASSURED AND ANNUITANTS.

WHOLE-LIFE PARTICIPATING ASSURANCES, MALES:—

I. AGGREGATE TABLES—O^{M} AND $O^{M(5)}$.

II. SELECT TABLES—$O^{[M]}$.

LIFE ANNUITANTS,

SELECT TABLES.

MALES ($O^{[am]}$) AND FEMALES ($O^{[af]}$).

(*Mortality Tables only.*)

BY

HENRY J. BAKER, F.I.A.

NOTE

(INSERTED ON THE AUTHORITY OF THE JOINT COMMITTEE)

AS TO

NET PREMIUMS FOR CONTINGENT SURVIVORSHIP ASSURANCES.

The Net Single and Annual Premiums, computed by the method set out in the following pages, 173 to 175, and included on pages 102 to 109 of the Select Tables issued by the Committee, are based upon the assumption that the vitality of the assured life will correspond with that of a Select Life under the $O^{[M]}$ Table. The deduced premiums must therefore be regarded as *minimum* rates, subject to adjustment where it is considered that this standard is not applicable. The separate data for Contingent Assurances, tabulated on pages 194 to 197 of the volume of Unadjusted Data (Minor Classes of Assurance), indicate for assured lives in this class a mortality higher than that of the $O^{[M]}$ Table, and probably more closely approximating to that of the O^{NM} (Whole-Life Non-Participating) Table. Reference may be made in this connection to Mr. CHATHAM'S Paper "On Premiums deduced from the Mortality Experience of British Life Offices." (Transactions of the Faculty of Actuaries, vol. 1, No. 5, pages 125 to 130).

NOTES ON THE METHODS ADOPTED IN THE CONSTRUCTION OF MORTALITY AND MONETARY TABLES DERIVED FROM THE EXPERIENCE OF LIVES ASSURED AND ANNUITANTS.

WHOLE-LIFE PARTICIPATING ASSURANCES—MALES.

I. AGGREGATE TABLES—O^M AND $O^{M(5)}$.

(*a*) ELEMENTARY VALUES.

(1). From the graduated values of colog p_x, furnished by Mr. G. F. Hardy, the l_x columns were deduced, the radix of the O^M Table being taken as 100,000 living at age 10, and that of the $O^{M(5)}$ Table as 107,324 at the same age. The latter Table merges into the former after age 84, the mortality of the two being thereafter identical. These values of l_x, expressed to the nearest integer, form the basis of the tabulated functions.

(2). The usual methods were employed in the construction of the columns of $\log l_x$, colog l_x, d_x, $\log d_x$, p_x, $\log p_x$, colog p_x, q_x, and e_x.

(3). The force of mortality, μ_x, was computed for the O^M Table ages 10-84, from the formula $\frac{7(d_{x-1}+d_x)-(d_{x-2}+d_{x+1})}{12 l_x}$, l_x and d_x being calculated for ages 7 to 9 by means of the fundamental formulæ given on pages 1 and 105 of the O^M and $O^{M(5)}$ Tables.

(4). For the $O^{M(5)}$ Table throughout, and for the O^M Table from age 85, $\mu_x = A + Bc^x$ (*see* page 105 of the $O^{M(5)}$ Tables), and the calculations were accordingly made on that basis. These values of μ_x may differ slightly from those which would have been deduced from the tabulated l_x and d_x, but the difference has no practical effect upon the annuity, or other monetary values.

(*b*) MONETARY VALUES.

(5). The values of $\log D_x$ were found by adding $\log v^x$ to $\log l_x$, each being taken to seven decimal places, and also by means of the formula $\log D_x = \log D_{x+1} + \text{colog } vp_x$. Adjustments were made where necessary, to correct the error introduced by the continued addition of $\log v$ or colog v.

(6). Similar methods were followed in the construction of $\log C_x$, the formulæ being :—

$$\log C_x = \log v^{x+1} + \log d_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and } \log C_x = \log C_{x+1} + \operatorname{colog} v + \Delta \operatorname{colog} d_x \quad . \quad . \quad . \quad . \quad (2)$$

(7). The values of $\log D_x$ and $\log C_x$ at different rates of interest were compared by means of the formulæ :—

$$\Sigma_x^{\omega-1} \log D'_x = \Sigma_x^{\omega-1} \log D_x + [x + (x+1) + \ldots + (\omega - 1)](\log v' - \log v) \quad (3)$$

$$\Sigma_x^{\omega-1} \log C'_x = \Sigma_x^{\omega-1} \log C_x + [(x+1) + (x+2) + \ldots + (\omega)](\log v' - \log v) \quad (4)$$

thus, by the O^M Table :—

$$\Sigma_{10}^{102} \log D_{(3\%)} = 334{\cdot}14737, \quad \Sigma_{10}^{102} \log C_{(3\%)} = 191{\cdot}60057,$$

$$\text{while } \Sigma_{10}^{102} \log D_{(2\frac{1}{2}\%)} + 5208(\log v_{(3\%)} - \log v_{(2\frac{1}{2}\%)}) = 334{\cdot}14738,$$

$$\text{and } \Sigma_{10}^{102} \log C_{(2\frac{1}{2}\%)} + 5301(\log v_{(3\%)} - \log v_{(2\frac{1}{2}\%)}) = 191{\cdot}60054.$$

(8). To obtain D_x and C_x from $\log D_x$ and $\log C_x$, five-figure logarithms were used, interpolation being made for the sixth place in $\log D_x$ and $\log C_x$.

(9). The formula $R_x = v\mathbb{S}_x - \mathbb{S}_{x+1}$ was used to supply a final check upon the accuracy of the $\mathbb{N}_x$ and M_x columns, thus, by the O^M Table at 3 per cent., $R_{10} = 753594{\cdot}90$, while $v\mathbb{S}_{10} - \mathbb{S}_{11} = 753594{\cdot}25$.

(10). Two independent methods were also employed in the calculation of $\log a_x$, $\log A_x$, and $\log P_x$, thus :—

$$\log a_x = \log \mathbb{N}_{x+1} - \log D_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\log a_{x+1} = \log a_x + \Delta \log \mathbb{N}_{x+1} + \Delta \operatorname{colog} D_x \quad . \quad . \quad . \quad (6)$$

$$\log A_x = \log M_x - \log D_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\log A_{x+1} = \log A_x + \Delta \log M_x + \Delta \operatorname{colog} D_x \quad . \quad . \quad . \quad . \quad (8)$$

$$\log P_x = \log M_x - \log \mathbb{N}_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$\log P_{x+1} = \log P_x + \Delta \log M_x + \Delta \operatorname{colog} \mathbb{N}_x \quad . \quad . \quad . \quad . \quad (10)$$

(11). The formula $v\Sigma(1 + a_x) - \Sigma a_x = \Sigma A_x$ was used as a further check, thus, by the O^M Table at 3 per cent., $v\Sigma_{10}^{102}(1 + a_x) - \Sigma_{10}^{102} a_x = 57{\cdot}928$, and $\Sigma_{10}^{102} A_x = 57{\cdot}928$.

(12.) For the computation of the continuous functions $\bar{a}_x$, $\bar{A}_x$, and $\bar{P}_x$, we have—

$$\bar{a}_x = a_x + \tfrac{1}{2} - \tfrac{1}{12}(\mu_x + \delta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\bar{A}_x = 1 - \delta \bar{a}_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\bar{P}_x = \frac{1}{\bar{a}_x} - \delta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

(13). These calculations were performed in duplicate, and five significant figures were retained in each value of the curtate annuity employed. The arithmometer was used for finding $\overline{A}_x$ by the above formula.

(14). The values of temporary annuities were obtained by means of the arithmometer in the manner explained in Part II of the Institute Text-Book (*v.* pages 403–404). Verification values were calculated at decennial intervals from $a_{x\overline{n}|}=\frac{\mathbb{N}_{x+1}-\mathbb{N}_{x+n+1}}{\mathrm{D}_x}$; and, finally, the sum of the values at each age was compared with the value of the expression

$$\frac{(\omega-x)\mathbb{N}_{x+1}-\mathbb{S}_{x+1}}{\mathrm{D}_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

(15). For the calculation of the values of annuities on two joint lives based on the O^{M} Table, commutation columns were formed, and the annuities then computed from the formula

$$\log a_{xy}=\log \mathbb{N}_{x+1:y+1}-\log \mathrm{D}_{xy} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

(16). Annuities on two, three, and four joint lives of equal age were calculated for the $\mathrm{O}^{\mathrm{M}(5)}$ Table by means of Gauss's logarithms, the formulæ used being of the type

$$\log \mathbb{N}_{xx}=\log \mathrm{D}_{xx}+[t](\log \mathbb{N}_{x+1:x+1}-\log \mathrm{D}_{xx}) \quad . \quad . \quad . \quad (16)$$

(17). Columns of $\log \mathrm{D}_{xx}$, $\log \mathrm{D}_{xxx}$, and $\log \mathrm{D}_{xxxx}$, were first formed by the continued addition of $\log l_x$ to $\log \mathrm{D}_x$ (thus, $\log \mathrm{D}_{xx}=\log l_x+\log \mathrm{D}_x$, $\log \mathrm{D}_{xxx}=\log l_x+\log \mathrm{D}_{xx}$, $\log \mathrm{D}_{xxxx}=\log l_x+\log \mathrm{D}_{xxx}$), seven places of decimals being used, while for the purpose of verification, values of $\mathbb{N}_{xx}$, $\mathbb{N}_{xxx}$, and $\mathbb{N}_{xxxx}$, were formed at decennial intervals by summing the corresponding D column.

(18). The $\mathrm{O}^{\mathrm{M}(5)}$ Table having been constructed according to Makeham's first modification of Gompertz's law, it follows that

$$a_{x(x+h_1)(x+h_2)\ldots(x+h_{m-1})}=a_{(x+t)(x+t)(\text{to } m \text{ terms})},$$

where $$c^x+c^{x+h_1}+\ldots+c^{x+h_{m-1}}=mc^{x+t},$$

and hence $$t=\log\left[\frac{1+c^{h_1}+c^{h_2}+\ldots+c^{h_{m-1}}}{m}\right]\div\log c.$$

(19). The Tables of Uniform Seniority for two and three lives, given on pages 244—249 of the $\mathrm{O}^{\mathrm{M}(5)}$ Tables, were constructed on this basis, the formulæ being :—

$$t=\frac{\log\left[\frac{1+c^h}{2}\right]}{\log c}, \text{ and } t=\frac{\log\left[\frac{1+c^h+c^{h+k}}{3}\right]}{\log c}, \text{ respectively } . \quad (17)\ (18)$$

(20). In the case of the Table given on pages 250—251 for finding the value of t when $a_{x:x:x:x+k} = a_{x+t:x+t:x+t:x+t}$ we have $3c^x + c^{x+k} = 4c^{x+t}$, whence $t = \dfrac{\log\left[\dfrac{3+c^k}{4}\right]}{\log c}$ . . . . . (19)

II. SELECT TABLES—$O^{[M]}$.

(*a*.) ELEMENTARY VALUES.

(1). The column $\log l_{[x]+t}$ (deduced from the graduated values of colog $p_{[x]+t}$ set out on pages 164 and 165 of the present volume) was taken as the basis of the Tables derived from Select Data.

(2). The remaining mortality functions were obtained as follows:—

$l_{[x]+t}$ (expressed to three decimal places at the oldest ages) = antilog of $\log l_{[x]+t}$.

$q_{[x]+t} = 1 - p_{[x]+t}$

$\log d_{[x]+t} = \log l_{[x]+t} + \log q_{[x]+t}$

$\mu_{[x]+t} = A_t + B_t c^{x+t}$ (see page 1 of the $O^{[M]}$ Tables).

(*b*.) MONETARY VALUES.

(3). The values of $\log D_{[x]+t}$, to seven places of decimals, were found in duplicate, by adding $\log v^{x+t}$ to $\log l_{[x]+t}$. In taking out the natural numbers, five-figure logarithms were used, interpolation being made for the sixth place of decimals in the value of $\log D_{[x]+t}$.

(4). Annuity values at date of entry, $a_{[x]}$, were found by the subtraction of $\log D_{[x]}$ from $\log \mathbb{N}_{[x]+1}$, the corresponding antilogarithms being tabulated to five significant figures. The values of $A_{[x]}$ and $P_{[x]}$ were derived from $a_{[x]}$, as follows:—

$A_{[x]} = 1 - d(1 + a_{[x]})$. (The arithmometer being employed for $d(1 + a_{[x]})$).

$P_{[x]} = \dfrac{1}{1 + a_{[x]}} - d$. (Oakes's Reciprocals being employed for $\dfrac{1}{1 + a_{[x]}}$).

(5). The functions $C_{[x]+t}$ and $M_{[x]+t}$ were not computed, it being found that annual and single premiums deduced therefrom differed but slightly from those calculated from the annuity values, the difference seldom exceeding unity in the fifth place of decimals. Thus, summing the values at 3 per cent., all entry ages:—

$$\Sigma_{10}^{75}\left\{[1 - d(1 + a_{[x]})] - \frac{M_{[x]}}{D_{[x]}}\right\} = -\cdot00015.$$

$$\Sigma_{10}^{75}\left\{\left[\frac{1}{1 + a_{[x]}} - d\right] - \frac{M_{[x]}}{\mathbb{N}_{[x]}}\right\} = -\cdot000014.$$

(6). In the construction of the select values of temporary annuities, tables of the values of $\Sigma_{t=1}^{t=n} D_{[x]+t}$ for each age at entry were prepared, and the annuities calculated therefrom by means of logarithms, $\log a_{[x]\overline{n}|} = \log \Sigma_{t=1}^{t=n} D_{[x]+t} - \log D_{[x]}$ . . . . . (20)

(7). The values of select annuities on two joint lives of equal age were found by means of commutation columns $D_{[x][x]} = l_{[x]} D_{[x]}$, and $\mathbb{N}_{[x][x]} = \Sigma_{t=0}^{t=9} D_{[x]+t:[x]+t} + \mathbb{N}_{x+10:x+10}$. The annuity values for each tenth of a year of age were then found by interpolation, using first differences.

(8). The formulæ used in the calculation of single and annual premiums for contingent assurances were :—

$$\log D_{[x]+n:[y]+n} = [\log v^{\frac{x+n}{2}} + \log l_{[x]+n}] + [\log v^{\frac{y+n}{2}} + \log l_{[y]+n}] \quad . \quad . \quad . \quad . \quad (21)$$

$$\log C_{\frac{1}{[x]+n:[y]+n}} = [\log v^{\frac{x+n+1}{2}} + \log (l_{[x]+n} - l_{[x]+n+1})] + [\log v^{\frac{y+n+1}{2}} + \log \tfrac{1}{2}(l_{[y]+n} + l_{[y]+n+1})] \quad (22)$$

$$\mathbb{N}_{[x][y]} = D_{[x][y]} + D_{[x]+1:[y]+1} + \ldots + D_{[x]+4:[y]+4} + D_{[x]+5:y+5} + \ldots + D_{[x]+9:y+9} + \mathbb{N}_{x+10:y+10} \quad (23)$$

$$M_{\frac{1}{[x][y]}} = C_{\frac{1}{[x][y]}} + C_{\frac{1}{[x]+1:[y]+1}} + \ldots + C_{\frac{1}{[x]+4:[y]+4}} + C_{\frac{1}{[x]+5:y+5}} + \ldots + C_{\frac{1}{[x]+9:y+9}} + M_{\frac{1}{x+10:y+10}} \quad (24)$$

$$A_{\frac{1}{[x][y]}} = \frac{M_{\frac{1}{[x][y]}}}{D_{[x][y]}} \quad . \quad . \quad . \quad (25) \qquad P_{\frac{1}{[x][y]}} = \frac{M_{\frac{1}{[x][y]}}}{\mathbb{N}_{[x][y]}} \quad . \quad . \quad . \quad (26)$$

(9). The male life $[x]$ being on the basis of the O$^{[M]}$ Table,* and the female life $[y]$ on the basis of the O$^{[F]}$ Table, the select values merge in the ultimate values after ten and five years respectively.

(10). Each age of $[x]$ from 10 to 75 being combined with each quinquennial age of $[y]$ from 20 to the end of life, the tables give the single and annual premiums for all differences between $[x]$ and $[y]$ within these limits which are multiples of 5. Thus, $[x]-[y]=$ 0, 5, &c., 55, and $[y]-[x]=0$, 5, &c., 85.

(11.) The values of $(\log v^{\frac{x}{2}} + \log l_x)$ and $(\log v^{\frac{y}{2}} + \log l_y)$ were first calculated and tabulated on separate slips, and the ultimate values of $\log D_{xy}$ were then found by adding together the corresponding values of these expressions for all the required combinations of x and y; and, the antilogarithms having been taken out, $\mathbb{N}_{xy}$ was then obtained by summing D_{xy}. The ultimate values of $\log C_{\frac{1}{xy}}$ were computed in a similar manner by combining the appropriate values of $(\log v^{\frac{x+1}{2}} + \log d_x)$ and $[\log v^{\frac{y+1}{2}} + \log \frac{1}{2}\ (l_y + l_{y+1})]$.

* *See* Note on p. 168.

(12). To obtain $D_{[x][y]}$, $D_{[x]+1:[y]+1}$, &c., and $C_{\frac{1}{[x][y]}}$, $C_{\frac{1}{[x]+1:[y]+1}}$, &c., separate slips were prepared for each of the functions $[\log v^{\frac{x+n}{2}} + \log l_{[x]+n}]$, $[\log v^{\frac{y+n}{2}} + \log l_{[y]+n}]$, $[\log v^{\frac{x+n+1}{2}} + \log (l_{[x]+n} - l_{[x]+n+1})]$, and $[\log v^{\frac{y+n+1}{2}} + \log \frac{1}{2} (l_{[y]+n} + l_{[y]+n+1})]$, n having the values 0 to 9 inclusive for x, and 0 to 4 inclusive for y.

(13). Each slip contained five values of x or y arranged vertically in the following manner:—

n	$\log l_{[y]+n}$	$(2) + \log v^{\frac{y+n}{2}}$
(1)	(2)	(3)
$[y]=20$		
0	5·00000	4·87163
1	4·99882	·86403
.		
.		
.		
9		
$[y]=21$		
0	4·99710	4·86231
1	·99592	·85471
.		
.		
.		
9		

n	$l_{[x]+n} - l_{[x]+n+1}$	log (2)	$(3) + \log v^{\frac{x+n+1}{2}}$
(1)	(2)	(3)	(4)
$[x]=10$			
0	240	2·38021	2·30961
1	410	·61278	·53576
.			
.			
.			
9			
$[x]=11$			
0	241	2·38202	2·30500
1	409	·61172	·52828
.			
.			
.			
9			

n	$\frac{1}{2}[l_{[y]+n} + l_{[y]+n+1}]$	log (2)	$(3) + \log v^{\frac{y+n+1}{2}}$
(1)	(2)	(3)	(4)
$[y]=20$			
0	99,864·5	4·99941	4·86462
1	99,543	·99801	·85680
2	99,117	·99615	·84852
3	98,588·5	·99383	·83978
4	97,979	·99113	·83066
5	...	...	·82135
6	...	...	·81194
7	...	...	·80248
8	...	...	·79297
9	...	...	·78339
$[y]=21$			
0	99,200	4·99651	4·85530

(14). The values of $[\log v^{\frac{y+n}{2}} + \log l_{[y]+n}]$ and $[\log v^{\frac{y+n+1}{2}} + \log \frac{1}{2}(l_{[y]+n} + l_{[y]+n+1})]$, given in the last column of these slips, for $n=5$, 6, 7, 8 and 9, were taken from the previously prepared slips, referred to in § (11), containing the ultimate values of these functions.

(15). By adding together the appropriate values from these slips, $\log D_{[x]+n:[y]+n}$ and $\log C_{\frac{1}{[x]+n:[y]+n}}$ were found, and then their anti-logarithms, whence, by combination with the ultimate tables, $\mathbb{N}_{[x][y]}$ and $M_{[x]^{\frac{1}{2}}[y]}$ were finally obtained. The following example illustrates the mode of procedure.

$[x]=70:[y]=20$		
n	$\log D_{[x]+n:[y]+n}$	$\cdot 0001 D_{[x]+n:[y]+n}$
0	8·94085	87,267
1	·91215	81,686
2	·87874	75,638
3	·84085	69,319
4	·79837	62,859
5	·75124	56,395
6	·69935	50,044
7	·64243	43,897
8	·58015	38,032
9	·51213	32,518
	$\cdot 0001\ \mathbb{N}_{80.30}=$	130,221
	$\cdot 0001\ \mathbb{N}_{[70][20]}$	**727,876**

(16). From values of $D_{[x][y]}$, $\mathbb{N}_{[x][y]}$, and $M_{[x]^{\frac{1}{2}}[y]}$, thus deduced, the single and annual premiums were readily computed by means of five-figure logarithms.

(17). The calculations were throughout worked in duplicate, and, in fact, this course has been adopted in the computation of nearly all the tabulated functions.

LIFE ANNUITANTS— MALES—$O^{[am]}$, AND FEMALES—$O^{[af]}$.

Elementary Values.

(1). The graduated functions supplied by Mr. G. F. Hardy, and taken as the bases of the monetary and other values, were $\log l_{[x]+t}$ for the Male section (see pages 4 and 5 of the $O^{[am]}$ Tables) and $l_{[x]+t}$ for the Female section (see pages 44 and 45 of the $O^{[af]}$ Tables).

(2). The tabulated values of $\log l_{[x]+t}$ for Female Annuitants are, however, correct to five decimal places, the values from $[x]=80$ being derived directly from the formula employed in the graduation.

(3). In both the Male and the Female sections we have

$$d_{[x]+t}=l_{[x]+t}-l_{[x]+t+1}$$

$$\log p_{[x]+t}=\log l_{[x]+t+1}-\log l_{[x]+t}$$

$$q_{[x]+t}=1-p_{[x]+t}.$$

(4). The curtate expectations of life for Female Annuitants were calculated from the formula $e_{[x]+t}=\frac{(l_{[x]+t+1}+l_{[x]+t+2}+\&c.)}{l_{[x]+t}}$, the values of $l_{[x]+t+1}$, &c., being those given on pages 44 and 45 of the O[af] Tables.

(5). The same formula was used for the Male Annuitants, but the values of $l_{[x]+t+1}$ were the antilogarithms to five significant figures of $\log l_{[x]+t+1}$ as tabulated on pages 4 and 5 of the O[am] Tables. This course was adopted in order that the expectations of life might be consistent with the annuity values. Thus, $e_{103}=\frac{l_{104}}{l_{103}}=\frac{\cdot46131}{1\cdot2474}=\cdot370$ (*see* page 19 of the O[am] Tables); while

$$a_{103} \text{ at } 2\tfrac{1}{2} \text{ per cent.}=\frac{D_{104}}{D_{103}}=\frac{v^{104}\times\cdot46131}{v^{103}\times 1\cdot2474}=\cdot361$$

(*See* page 25 of the O[am] Tables.)

(6). For the Female section, $e_{103}=\frac{l_{104}}{l_{103}}=\frac{2}{5}=\cdot400$

(*See* page 61 of the O[af] Tables),

and $$a_{103} \text{ at } 2\tfrac{1}{2} \text{ per cent.}=\frac{D_{104}}{D_{103}}=\frac{v^{104}\times 2}{v^{103}\times 5}=\cdot390$$

(*See* page 67 of the O[af] Tables).

(7). The mortality of the Male Annuitants following Makeham's Law, a table of Uniform Seniority was prepared from the formula $t=\frac{\log\left[\frac{1+c^k}{2}\right]}{\log c}$ (since $c^x+c^{x+k}=2c^{x+t}$). The table is applicable at all ages, all rates of interest, and for all durations from the date of purchase of the annuity, thus, $a_{[x-n]+n:[x-n+k]+n}=a_{[x-n+t]+n:[x-n+t]+n}$.

(8). The tables of $a_{[xx]}$ and of a_{xx}, given on pages 192–227 of the O[am] and O[af] Tables, for each tenth of a year of age, were interpolated by first differences from the values computed at yearly intervals of age.

HENRY J. BAKER.

NOTES ON THE METHODS ADOPTED

IN THE

CALCULATION OF THE MONETARY TABLES,

DERIVED FROM THE

ANNUITANT EXPERIENCE.

SELECT TABLES,

MALES ($O^{[am]}$) AND FEMALES ($O^{[af]}$).

I. SINGLE LIVES.

II. JOINT LIVES.

BY

JAMES CHATHAM, F.I.A., F.F.A.

DESCRIPTION OF THE METHOD ADOPTED IN THE CALCULATION OF THE MONETARY TABLES DERIVED FROM THE ANNUITANT EXPERIENCE.

I.—Single Lives.

(9*). The values required for males and females were:

(*a*) $D_{[x]+t}$, $\mathbb{N}_{[x]+t}$, $a_{[x]+t}$ for all ages and all values of t from 0 to 5, at $2\frac{1}{2}$, 3, and $3\frac{1}{2}$ per cent.

(*b*) $a_{[x]}$ and a_x for all ages at $2\frac{1}{4}$, $2\frac{3}{4}$, $3\frac{1}{4}$, 4, $4\frac{1}{2}$, and 5 per cent.

The basis for the calculations in the case of males was $\log l_{[x]+t}$, and in the case of females $l_{[x]+t}$.

(10). **Ultimate Values.**—The values of $\log D_x$ were obtained by adding $\log v^x$ to $\log l_x$, five-place logarithms being used. The natural numbers to 5 significant figures were taken out, and continuous summations gave $\mathbb{N}_x$. The logarithms of $\mathbb{N}_{x+1}$ to 5 places were next extracted, and $\log D_x$ deducted, giving $\log a_x$, from which a_x to three decimal places was in turn obtained.

(11). **Select Values.**—A similar process was followed in obtaining the values of $D_{[x]+4}$, $D_{[x]+3}$, $D_{[x]+2}$, $D_{[x]+1}$. They were then added successively to $\mathbb{N}_{x+5}$, thus forming $\mathbb{N}_{[x]+4}$, $\mathbb{N}_{[x]+3}$, $\mathbb{N}_{[x]+2}$, $\mathbb{N}_{[x]+1}$. The logarithms to five places were next taken out, and $\log D_{[x]+4}$, $D_{[x]+3}$, $D_{[x]+2}$, $D_{[x]+1}$, $D_{[x]}$ subtracted. This gave $\log a_{[x]+4}$, $a_{[x]+3}$, $a_{[x]+2}$, $a_{[x]+1}$, $a_{[x]}$, and the natural numbers were extracted to three decimal places. For some rates of interest intermediate annuity values were not required, and in these cases continuous summation of $D_{[x]+t}$ was dispensed with.

The work was done in duplicate, and the results compared.

II.—Joint Lives.

(12). It was desired to calculate the values of Joint Life Annuities for all combinations of two lives for equal ages and for quinquennial differences in age at $2\frac{1}{2}$, 3, and $3\frac{1}{2}$ per cent., and for equal ages only at 4 and 5 per cent., all values being obtained at date of purchase and 5 years after.

(13). There are various methods of calculating the values of these Annuities; but perhaps the one most generally adopted is that advocated in the *Institute of Actuaries' Text-Book*. That method, however, did not seem suitable in the present case, because it necessitates a calculation of values for all combinations of ages; and, as stated above, the values were required for quinquennial differences in age only. Another reason why it did not seem suitable is that

* For §§ (1) to (8), dealing with the Elementary values in the Annuitant Experience, *see* pp. 175, 176.

the work was to be spread over a considerable number of computers; and as some of them had no knowledge of actuarial science, it was desirable that the method adopted should be as simple as possible. What I may call the "Slip Method," as used by the late Mr. Chisholm, was, therefore, decided upon.

(14). **Ultimate Values.**—The values, according to the ultimate rate of mortality, were calculated first of all, and accordingly slips of $\log l_x$ and $\log D_y$ at $2\frac{1}{2}$, 3, and $3\frac{1}{2}$ per cent. were formed for both males and females. These were done in duplicate, and printed to facilitate the work. The necessary values of $\log D_{xy}$ in Davies' form were then formed, and from these values were successively obtained, D_{xy}, $\mathbb{N}_{xy}$, $\log \mathbb{N}_{xy}$, $\log a_{xy}$, and a_{xy}. A specimen of the actual working with relative slips is appended.

(15). **Select Values.**—A similar method was followed in calculating these, the only difference being that 5 preparatory columns were required instead of one. Slips were formed in duplicate of the logs of $l_{[x]}$, $l_{[x]+1}$, $l_{[x]+2}$, $l_{[x]+3}$, $l_{[x]+4}$, and of $D_{[y]}$, $D_{[y]+1}$, $D_{[y]+2}$, $D_{[y]+3}$, $D_{[y]+4}$, and the values of the logs of $D_{[xy]}$, $D_{[xy]+1}$, $D_{[xy]+2}$, $D_{[xy]+3}$, $D_{[xy]+4}$ obtained. The natural numbers of these values, with the exception of the first, were taken out, and then added to $\mathbb{N}_{[xy]+5}$, which gave $\mathbb{N}_{[xy]+1}$. The work then proceeded as before. A specimen of the actual working in this case also, with relative slips, is appended.

(16). Before commencing the actual calculations, trials were made; and as the values by 5 place logarithms were found to be practically identical with those by 6 place logarithms, the former were throughout adopted.

(17). The whole of the work was done in duplicate, and the two sets were compared at three stages—

1st. The ultimate D_{xy}.
2nd. The ultimate a_{xy}.
3rd. The select $a_{[xy]}$.

In addition, values at intervals were checked by means of the approximate summation formula No. 33 in the *Text-Book*, and the results, after deducting the rough adjustment of ·5, agreed very closely with the original values. In the majority of cases the difference was only ·001, the greatest difference being ·004.

(18). I take this opportunity of acknowledging my indebtedness to Mr. Alexander Fraser, F.I.A., F.F.A., of the Scottish Life Assurance Company, Limited, for the valuable assistance he has rendered me in connection with the work.

JAMES CHATHAM.

MALE ULTIMATE—2½ per cent.

log l_x and log D_y.

x	log l_x
25	4·98986
26	·98679
27	·98368
28	·98052
29	·97732 ·91817
30	·97406
31	·97073
32	·96734
33	·96388
34	·96034 ·75452
...	...
...	...
...	...
95	2·48089
96	·26504
97	·02970
98	1·77307
99	·49321 ·96212
100	·18801
101	0·85513
102	·49205
103	·09601
104	$\bar{1}$·66399
	·25731

y	log D_y
25	4·72176
26	·70797
27	·69414
28	·68025
29	·66633 ·47045
30	·65234
31	·63829
32	·62418
33	·60999
34	·59573 ·59098
...	...
...	...
...	...
95	1·46212
96	·23555
97	0·98949
98	·72213
99	·43155 ·09615
100	·11562
101	$\bar{1}$·77202
102	·39822
103	$\bar{2}$·99145
104	·54871
	·92217

The figures printed in small black type represent the sum of all the preceding values.

VALUE OF A JOINT LIFE ANNUITY OF £1.—ULTIMATE.

x and y BOTH MALE.

Rate per cent. 2½. Difference in age 5 years.

x (1)	y (2)	log D_{xy} (3)	D_{xy} (4)	N_{xy} (5)	log $N_{x+1:y+1}$ (6)	(6)−(3) (7)	a_{xy} (8)
25	30	9·64220	43873	84655984849·8329	10·90455	1·26235	18·296
26	31	62508	42177	802686	88110	25602	18·031
27	32	60786	40538	760509	85731	24945	17·760
28	33	59051	38950	719971	83316	24265	17·484
29	34	57305	37415	681021	80862	23557	17·202
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...
95	100	2·59651	394·92	537·53	2·15415	$\bar{1}$·55764	·361
96	101	03706	108·91	142·61	1·52767	49061	·309
97	102	1·42792	26·787	33·702	0·83985	41193	·258
98	103	0·76452	5·8146	6·9159	·04191	27739	·189
99	104	04192	1·1013	1·1013	...	...	...
		·41384	84655984849·8329	...	·85392	·48200	...

MALE—SELECT.

	$\log l_{[x]}$	$\log l_{[x]+1}$	$\log l_{[x]+2}$	$\log l_{[x]+3}$	$\log l_{[x]+4}$
20	5·00000	4·99886	4·99733	4·99530	4·99277
21	4·99708	·99592	·99437	·99230	·98974
22	·99412	·99294	·99137	·98928	·98667
23	·99113	·98994	·98833	·98621	·98356
24	·98811 ·97044	·98689 ·96455	·98526 ·95866	·98309 ·94618	·98041 ·93315
25	·98505	·98380	·98214	·97993	·97720
26	·98195	·98067	·97897	·97672	·97394
27	·97880	·97749	·97575	·97346	·97061
28	·97560	·97426	·97248	·97013	·96722
29	·97234 ·86418	·97097 ·85174	·96914 ·83514	·96674 ·81316	·96376 ·78588
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
90	3·19540	3·10735	2·99194	2·84788	2·67635
91	·04410	2·94809	·82224	·66517	·47815
92	2·87920	·77450	·63726	·46599	·26206
93	·69947	·58528	·43561	·24883	·02645
94	·50354 ·54686	·37900 ·00252	·21575 ·97805	·01205 ·48958	1·76954 ·41279
95	·28993	·15409	1·97603	1·75386	·48937
96	·05702	1·90885	·71463	·47230	·18382
97	1·80306	·64142	·42956	·16523	0·85057
98	·52611	·34978	·11866	0·83032	·48709
99	·22407	·03171	0·77957	·46503	·09060
	·44705	·08837	·99450	·12630	·51424

The figures printed in small black type represent the sum of all the preceding values.

MALE—SELECT—2½ per cent.

	log $D_{[y]}$	log $D_{[y]+1}$	log $D_{[y]+2}$	log $D_{[y]+3}$	log $D_{[y]+4}$
20	4·78552	4·77366	4·76140	4·74865	4·73540
21	·77188	·75999	·74772	·73493	·72164
22	·75819	·74629	·73400	·72118	·70785
23	·74448	·73257	·72023	·70739	·69402
24	·73074	·71879	·70644	·69355	·68014
	·79081	·73130	·66979	·60570	·53905
25	·71695	·70498	·69260	·67966	·66621
26	·70313	·69113	·67870	·66573	·65222
27	·68926	·67722	·66476	·65174	·63817
28	·67533	·66327	·65076	·63769	·62406
29	·66135	·64925	·63670	·62358	·60987
	·23683	·11715	·99331	·86410	·72958
...	...	...	...	...	...
...	...	...	...	...	...
...	...	...	...	...	...
90	2·23025	2·13148	2·00534	1·85056	1·66831
91	·06823	1·96149	1·82492	·65713	·45938
92	1·89260	·77718	·62922	·44722	·23257
93	·70215	·57724	·41684	·21934	0·98624
94	·49550	·36023	·18626	0·97184	·71860
	·70233	·35370	·52294	·18217	·35111
95	·27116	·12460	0·93582	·70292	·42771
96	·02753	0·86864	·66369	·41064	·11143
97	0·76285	·59048	·36790	·09284	$\bar{1}$·76746
98	·47517	·28812	·04627	$\bar{1}$·74721	·39326
99	·16241	$\bar{1}$·95932	1·69646	·37120	$\bar{2}$·98604
	·40145	·18486	·23308	·50698	·03701

The figures printed in small black type represent the sum of all the preceding values.

VALUE OF A JOINT-LIFE ANNUITY OF £1—SELECT.

Rate per cent. $2\frac{1}{2}$. x and y BOTH MALE. Difference in age 5 years.

x (1)	y (2)	log $D_{[xy]}$ (3)	log $D_{[xy]+1}$ (4)	log $D_{[xy]+2}$ (5)	log $D_{[xy]+3}$ (6)	log $D_{[xy]+4}$ (7)	$D_{[xy]+1}$ (8)	$D_{[xy]+2}$ (9)	$D_{[xy]+3}$ (10)	$D_{[xy]+4}$ (11)	$N_{[xy]+5}$ (12)	$\Sigma[8]$ to $[12] = N_{[xy]+1}$ (13)	log [13] (14)	[14]−[3] (15)	$a_{[xy]}$ (16)
20	25	9·71695	9·70384	9·68993	9·67496	9·68598	50564	48970	47311	45602	846560	1039007	11·01662	1·29967	19·937
21	26	·70021	·68705	·67307	·65803	·64196	48646	47105	45502	43849	802687	987789	10·99466	·29445	19·699
22	27	·68338	·67016	·65613	·64102	·62484	46791	45303	43754	42154	760510	938512	·97244	·28906	19·456
23	28	·66646	·65321	·63909	·62390	·60762	45000	43560	42063	40515	719972	891110	·94993	·28347	19·207
24	29	·64946	·63614	·62196	·60667	·59028	43265	41876	40427	38930	681022	845520	·92712	·27766	18·952
..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
90	95	4·46656	4·23195	3·92776	3·55080	3·10406	17059	8467·6	3554·7	1270·8	537·5	30889·6	4·48982	0·03326	1·055
91	96	·07163	3·81673	·48593	·07581	2·58958	6557·4	3061·5	1190·7	388·67	142·61	11340·88	·05465	$\bar{1}$·98302	·961
92	97	3·64205	·36498	·00516	2·55883	·02952	2317·3	1012·0	362·10	107·03	33·70	3832·13	3·58344	·94139	·874
93	98	·17464	2·87340	2·48188	1·99604	1·41971	747·14	303·31	99·092	26·285	6·916	1182·743	·07287	·89823	·791
94	99	2·66595	·33832	1·91221	·38325	0·75558	217·93	81·698	24·169	5·6961	1·1013	330·5944	2·51929	·85334	·713
		·15750	·45608	·53934	·34084	·91075	10204280982 9·77	9794783413 2·108	9362737559 6·661	8925548717 5·0811	1265606837 675·8273	1648480344409·4474	·22872	·07122	..

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INDEX I.—COMPILATION OF DATA (pp. 1–120) *continued.*

INDEX I.—COMPILATION OF DATA (pp. 1–120) *continued.*

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$=\alpha+\beta c^x$	134	(2)
$\mu_x=-\frac{d}{dx}\log_e l_x=\frac{1}{M}\left(\alpha+\beta\frac{\log_e c}{c-1}\cdot c^x\right)$		
$=A+Bc^x$	134	(3)
$\log_{10}l_{[x]+t}=(\log_{10}k-f_t)-(x+t)\alpha-\beta\left(\frac{1}{c-1}+\frac{\psi_t}{c^t}\right)c^{x+t}$		
$=\log_{10}k_t+(x+t)\log_{10}s+\log_{10}g_t c^{x+t}$ . .	134	(4)
$\operatorname{colog}_{10}(p_{[x]+t})=-\Delta_t\log_{10}l_{[x]+t}$		
$=(\alpha+\Delta f_t)+\beta\left(1+\frac{\Delta\psi_t}{c^t}\right)c^{x+t}$		
$=\alpha_t+\beta_t c^{x+t}$	134	(5)
$\mu_{[x]+t}=-\frac{d}{dt}\log_e l_{[x]+t}$		
$=\frac{1}{M}\left[\left(\alpha+\frac{d}{dt}f_t\right)+\beta\left(\frac{\log_e c}{c-1}+\frac{1}{c^t}\cdot\frac{d}{dt}\psi_t\right)c^{x+t}\right]$		
$=A_t+B_t c^{x+t}$	134	(6)
$\log_{10}l_{[x]+t}=\log_{10}l_{x+t}-f_t-\beta c^x\psi_t$	135	(7)
$\operatorname{colog}_{10}(p_{[x]+t})=\operatorname{colog}_{10}(p_{x+t})+\Delta f_t+\beta c^x\Delta\psi_t$ . . .	135	(8)
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$a^{YY}_{yz}=\frac{a_{y_1z_1}+r_y a_{y_2z_1}+r_z a_{y_1z_2}+r_y r_z a_{y_2z_2}}{(1+r_y)(1+r_z)}$ . .	137	(15)
$a^{MYY}_{xyz}=\frac{a_{xy_1z_1}+r_y a_{xy_2z_1}+r_z a_{xy_1z_2}+r_y r_z a_{xy_2z_2}}{(1+r_y)(1+r_z)}$.	137	(16)

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$\phi_x=\cdot000050400(10)^{-\cdot0032(29-x)^2}+\cdot000011385(10)^{-\cdot0060(65\cdot5-x)^2}$	151	(20)
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$= \log C_{x+1} + \operatorname{colog} v + \Delta \operatorname{colog} d_x$ . . .	170	(2)
$\Sigma_x^{\omega-1} \log D'_x = \Sigma_x^{\omega-1} \log D_x + [x + (x+1) + \ldots$ $\ldots + (\omega-1)](\log v' - \log v)$	170	(3)
$\Sigma_x^{\omega-1} \log C'_x = \Sigma_x^{\omega-1} \log C_x + [(x+1) + (x+2) + \ldots$ $\ldots + (\omega)](\log v' - \log v)$	170	(4)
$\Sigma_x^{\omega-1} M_x = R_x = v\Sigma_x^{\omega-1} \mathbb{N}_x - \Sigma_{x+1}^{\omega-1} \mathbb{N}_x = v\mathbb{S}_x - \mathbb{S}_{x+1}$ . .	170	—
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$\log A_{x+1} = \log A_x + \Delta \log M_x + \Delta \operatorname{colog} D_x$ . . .	170	(8)
$\log P_x = \log M_x - \log \mathbb{N}_x$	170	(9)
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$\overline{A}_x = 1 - \delta \bar{a}_x$	170	(12)
$\overline{P}_x = \frac{1}{\bar{a}_x} - \delta$	170	(13)
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Three Lives: $a_{x:x+h:x+h+k}=a_{x+t:x+t:x+t}$		
$t=\dfrac{\log\left[\dfrac{1+c^h+c^{h+k}}{3}\right]}{\log c}$	171	(18)
Four Lives: $a_{x:x:x:x+k}=a_{x+t:x+t:x+t:x+t}$		
$t=\dfrac{\log\left[\dfrac{3+c^k}{4}\right]}{\log c}$	172	(19)
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$\mathrm{P}_{[x]}=\dfrac{1}{1+a_{[x]}}-d$	172	—
$\log a_{[x]\overline{n}|}=\log \sum_{t=1}^{t=n} \mathrm{D}_{[x]+t}-\log \mathrm{D}_{[x]}$ . .	173	(20)
$\mathrm{D}_{[x][x]}=l_{[x]}\mathrm{D}_{[x]}$	173	—
$\mathbb{N}_{[x][x]}=\sum_{t=0}^{t=9} \mathrm{D}_{[x]+t:[x]+t}+\mathbb{N}_{x+10:x+10}$ . .	173	—
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$\log \mathrm{D}_{[x]+n:[y]+n}=[\log v^{\frac{x+n}{2}}+\log l_{[x]+n}]+[\log v^{\frac{y+n}{2}}+\log l_{[y]+n}]$.	173	(21)
$\log \mathrm{C}_{\frac{1}{[x]+n:[y]+n}}=[\log v^{\frac{x+n+1}{2}}+\log (l_{[x]+n}-l_{[x]+n+1})]$ $+[\log v^{\frac{y+n+1}{2}}+\log \frac{1}{2}(l_{[y]+n}+l_{[y]+n+1})]$	173	(22)
$\mathbb{N}_{[x][y]}=\mathrm{D}_{[x][y]}+\mathrm{D}_{[x]+1:[y]+1}+\ldots+\mathrm{D}_{[x]+4:[y]+4}$ $+\mathrm{D}_{[x]+5:y+5}+\ldots+\mathrm{D}_{[x]+9:y+9}+\mathbb{N}_{x+10:y+10}$	173	(23)
$\mathrm{M}^{1}_{[x][y]}=\mathrm{C}^{1}_{[x][y]}+\mathrm{C}_{\frac{1}{[x]+1:[y]+1}}+\ldots+\mathrm{C}_{\frac{1}{[x]+4:[y]+4}}$ $+\mathrm{C}_{\frac{1}{[x]+5:y+5}}+\ldots+\mathrm{C}_{\frac{1}{[x]+9:y+9}}+\mathrm{M}_{\frac{1}{x+10:y+10}}$	173	(24)
$\mathrm{A}^{1}_{[x][y]}=\dfrac{\mathrm{M}^{1}_{[x][y]}}{\mathrm{D}_{[x][y]}}$	173	(25)
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ANNUITY VALUES ON TWO LIVES OF EQUAL AGE:—		
Interpolation formula for tenths of a year of age:—		
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$\log a_x=\log N_{x+1}-\log D_x$	179	—
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$N_{[x]+1}=D_{[x]+1}+D_{[x]+2}+\ldots+D_{[x]+4}+N_{x+5}$.	179	—
$\log a_{[x]+t}=\log N_{[x]+t+1}-\log D_{[x]+t}$	179	—
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$\log D_{xy}=\log l_x+\log D_y$ (where x is not $>y$) . .	180	—
$\log a_{xy}=\log N_{x+1:y+1}-\log D_{xy}$	180	—
$\log D_{[x]+t:[y]+t}=\log l_{[x]+t}+\log D_{[y]+t}$ (where x is not $>y$) .	180	—
$N_{[x]+1:[y]+1}=D_{[x]+1:[y]+1}+D_{[x]+2:[y]+2}+\ldots$ $\ldots+D_{[x]+4:[y]+4}+N_{x+5:y+5}$	180	—
$\log a_{[x]+t:[y]+t}=\log N_{[x]+t+1:[y]+t+1}-\log D_{[x]+t:[y]+t}$ . .	180	—